Reports of the Department of Geodetic Science

Report No. 211

PROCEDURES AND RESULTS RELATED TO THE DIRECT DETERMINATION OF GRAVITY ANOMALIES FROM SATELLITE AND TERRESTRIAL GRAVITY DATA

by

Richard H. Rapp

Prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland 20770

Grant No. NGR 36-008-161
OSURF Project No. 3210

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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Abstract

The equations needed for the incorporation of gravity anomalies as unknown parameters in an orbit determination program are described. These equations were implemented in the Geodyn computer program which was then used to process optical satellite observations. Besides the arc dependent parameters unknowns, we consider 184 15° unknown anomalies and coordinates of 7 tracking stations. Up to 39 arcs (5 - 7 day) involving 10 different satellites, were processed. An anomaly solution just from the satellite data and a combination solution with 15° terrestrial anomalies was made. The results with the somewhat limited data sample indicate that the method works. The report gives the 15° anomalies from various solutions and the potential coefficients implied by the different solutions.

Foreword

This report was prepared by Richard H. Rapp, Professor, Department of Geodetic Science, The Ohio State University, under NASA Grant NGR36-008-161, The Ohio State University Research Foundation Project No. 3210. The contract covering this research is administered through the Goddard Space Flight Center, Greenbelt, Maryland, Dr. David E. Smith, Technical Officer.

The author is indebted to Mr. Pentti Karki who carried out the modifications to the Geodyn program and who made most of the computer runs required for this report. In addition Mr. Tom Martin, of the Wolf Research and Development Corporation provided valuable assistance in answering our questions about the Geodyn program. Mr. D. P. Hajela prepared the terrestrial gravity material needed for this study and some other data analysis programs. Some computer time that was not supported through the project was provided by the Instruction and Research Computer Center of The Ohio State University.

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1. Introduction

The gravity field of the earth may be represented in several ways. Among them are through potential coefficients ($\overline{C}_{\ell m}$, $\overline{S}_{\ell m}$) discrete mean gravity anomalies, (Δg), and discrete surface density values (χ). Each of these representations has its advantages and disadvantages. In describing the orbital motion of satellites the use of potential coefficients is most convenient. The use of mean gravity anomalies or mean surface density values allows the incorporation of discrete blocks on the surface of the earth into the gravitational model. Such a representation may be useful as a procedure independent of potential coefficient determination, or in the analysis of the gravitational field in local areas that may be obtained by precise satellite observations as may be obtained from satellite-to-satellite tracking, laser range measurements, or altimeter measurements.

Arnold (1965, 1966) suggested that discrete anomalies could be found in selected areas by analyzing the change of satellite orbital elements. The procedures of Arnold have been described in several articles by he and his colleagues, the latest of which is Arnold (1972) where he analyzed 1182 error equations to solve for 52, $20^\circ \times 20^\circ$ anomalies.

Koch (1968) proposed a solution where the gravitational field is described by a set of low degree potential coefficients and a set of discrete surface densities distributed on the surface of the earth. Koch and Morrison (1970) gave the first results from this new method, analyzing optical satellite observations from four satellites. In their computations they used a low degree field to degree four plus 48, $30^\circ \times 30^\circ$ density values. Additional work in this direction was reported by Koch and Witte (1971) where they used ten weeks of Doppler data from five satellites to determine the coordinates of 27 tracking stations and density values for 104, 20° surface elements. Koch (1974) reports results with additional Doppler data solving for 104 density values, 123 station coordinates and additional arc dependent parameters.

Rapp (1967) extended in a general way the ideas of Arnold to show how a global solution for discrete anomalies could be made. This paper was extended further by Obenson (1970) who worked out equations needed for one type of discrete solution and carried out simulation studies to verify the equations and method. Rapp (1971a) published another theoretical approach to the direct recovery of gravity anomalies from the analysis of satellite data and carried out simulation studies to verify the method. Haverland (1971) carried out an extensive analysis of certain equations needed in the direct recovery procedure. Finally, Rapp (1971b) discussed the procedures to be actually used in carrying out a solution for discrete anomalies and the combination with existing terrestrial gravity material. This report presents results for determining discrete anomalies using satellite and terrestrial data based on the suggestions of Rapp (1971b).

2. Basic Method and Adjustment Procedure.

The basic method used in this study consists of the numerical integration of the equations of motions of the satellite considering all pertinent forces acting on the satellite and the development of observation equations through the integration (simultaneously with the orbit integration) of the variational equations which will be a function of the unknowns to be solved for.

The gravitational field of the earth is represented by a set of potential coefficients (which are used for reference purposes only and thus are regarded fixed) and by a set of mean gravity anomalies. (For this report we used 184, 15° equal area mean gravity anomalies. Conceptually smaller blocks could also be used.) Thus the gravitational field is represented by:

$$V = U + T \tag{1}$$

where V is the total gravitational potential, U is the gravitational potential due to a set of reference potential coefficients, and T is the disturbing potential with respect to U, formulated as a function of the mean gravity anomalies. We have:

$$U = \frac{kM}{r}\left[1 + \sum_{\ell=2}^{\infty}\left(\frac{a}{r}\right)^{\ell}\sum_{m=0}^{\ell}[\overline{C}_{\ell m}\cos m\lambda + \overline{S}_{\ell m}\sin m\lambda]\,\overline{P}_{\ell m}(\sin\varphi')\right] \tag{2}$$

and

$$T = \frac{R}{4\pi}\iint_{\sigma} \Delta g'\, S(r,\psi)\, d\sigma \tag{3}$$

where

$\overline{C}_{\ell m}$, $\overline{S}_{\ell m}$ are fully normalized potential coefficients;
r is the distance from the center of earth to the satellite;
ψ is the spherical arc between the element $d\sigma$ on the surface of the earth and the subsatellite point;
$S(r, \psi)$ is the generalized Stokes' function (Heiskanen and Moritz, 1967); (see equation 20 of this report);
$\Delta g' = \Delta g_T - \Delta g_{PC}$ where Δg_T are terrestrial anomalies referred to some gravity formula and Δg_{PC} are the anomalies implied by the potential coefficients used in (2).

An observation, $\overline{O}$, may be represented as a function as follows:

$$\overline{O}\,(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0, t_0, t, N, p_1, p_2 \ldots p_i, \Delta g_1', \Delta g_2', \ldots \Delta g_n', X_s, Y_s, Z_s) = 0 \tag{4}$$

where $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$ are the initial position and velocity terms at an epoch t_0; t is the time of the observations; N is a set of reference potential coefficients; p_i are parameters related to radiation pressure, air drag, etc.; the $\Delta g'$ values are the unknown anomalies to be solved for; and X_s, Y_s, Z_s are the observation station coordinates. Considering only those quantities that may be solved for in an adjustment with satellite data we write (4) as:

$$\overline{O}\,(\underline{r}, \underline{\dot{r}}, \underline{p}, \underline{\Delta g'}, \underline{X}) = 0 \tag{5}$$

The observation equation is formed as:

$$\underline{\Delta O} = \frac{\partial O}{\partial \underline{r}} \left(\frac{\partial \underline{r}}{\partial \underline{r_0}} \underline{\Delta r_0} + \frac{\partial \underline{r}}{\partial \underline{\dot{r}_0}} \underline{\Delta \dot{r}_0} + \frac{\partial \underline{r}}{\partial \underline{\Delta g}} \Delta(\underline{\Delta g'}) + \frac{\partial \underline{r}}{\partial \underline{p}} \underline{\Delta p} + \frac{\partial \underline{r}}{\partial \underline{X}} \Delta \underline{X} \right) \tag{6}$$

where, in the case of this report the observation will be declination or right ascension only.

In more general terms we can express equation (5) as:

$$F\,(L_{F^a}, L_{X^a}) = 0 \tag{7}$$

where F is the observation function, L_{F^a} is a vector of adjusted observations and L_{X^a} is a vector of adjusted parameters. As can be seen from (6) the parameters considered in this problem are $\underline{r_0}$, $\underline{\dot{r}_0}$, $\underline{\Delta g'}$, $\underline{p}$, and $\underline{X}$ values.

In carrying out the adjustment where the anomalies are the unknowns, we must subject the anomalies to certain conditions. These conditions are that the

$a_{1,0}$, $a_{1,1}$, $b_{1,1}$, $a_{2,1}$, $b_{2,1}$ coefficients in the spherical harmonic expansion of the adjusted anomalies are zero and that the $a_{0,0}$ term is either zero or some defined value. Such a procedure is analogous in the usual solution for potential coefficients where the first degree and the $\overline{C}_{2,1}$, $\overline{S}_{2,1}$ coefficients are set to zero. The conditions may be written as:

$$G(L_{X^a}) = 0 \tag{8}$$

where the term in L_{X^a} in (8) refers only to the $\underline{\Delta g}'$. (See section 5 for a detailed explanation of the anomaly spherical harmonic coefficients and the elements of the G matrix given in equation (8)).

The linearized form of equations (7) and (8) are:

$$\begin{aligned} B_F V_F + B_{FX} V_X + W_F &= 0 \\ B_{GX} V_X + W_G &= 0 \end{aligned} \tag{9}$$

If we let P_F be the weight matrix for the observations and P_X be the a priori weight matrix for observed values of the parameters ($\underline{\Delta g}'$ in this case) we have (Mikhail, 1970):

$$\begin{bmatrix} V_X \\ -K_G \end{bmatrix} = \begin{bmatrix} s^2 B'_{FX} P_F B_{FX} + P_X & B'_{GX} \\ B_{GX} & 0 \end{bmatrix}^{-1} \begin{bmatrix} -s^2 B'_{FX} P_F W_F + P_X W_X \\ -W_G \end{bmatrix} \tag{10}$$

where W_X is the difference between the observed value of a parameter and the approximate value used in computing the W_F misclosure and W_G is evaluated using (8) with the approximate values of the parameters. s^2 is a scaling parameter that permits a proper balance between satellite solutions for anomalies and the terrestrial gravity data. In a solution without terrestrial data the s^2 value has no effect on the solution. In the form (10) is now written the direct combination of gravimetric and satellite data can be carried out. If a solution for anomalies from satellite data is desired, it is only necessary to set P_X and W_G (except for the Δg_0 term) equal to zero.

In practice, the form of the equations in (10) allows the elimination of arc dependent quantities after the processing of each arc so that the main unknowns to be solved for are the station coordinates and anomalies.

3. Gravitational Force Components From Potential Coefficients and Gravity Anomalies.

The main force acting on the satellite are the gravitational forces. For most studies dealing with the determination of satellite orbits and the determination of the earth's gravitational field, the gravitational potential has been represented by the U term of equations (1) and (2). Now we need to incorporate the gravity anomalies (in T through equation (3)) in the force model computations.

To start we note that the gravitational forces acting on the satellite may be found by differentiating the gravitational potential. The accelerations in the x, y, z true of date coordinate system can then be written as:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial x} & \frac{\partial \varphi'}{\partial x} & \frac{\partial \lambda}{\partial x} \\ \frac{\partial r}{\partial y} & \frac{\partial \varphi'}{\partial y} & \frac{\partial \lambda}{\partial y} \\ \frac{\partial r}{\partial z} & \frac{\partial \varphi'}{\partial z} & \frac{\partial \lambda}{\partial z} \end{bmatrix} \begin{bmatrix} \frac{\partial V}{\partial r} \\ \frac{\partial V}{\partial \varphi'} \\ \frac{\partial V}{\partial \lambda} \end{bmatrix} \tag{11}$$

where the partial derivatives may be found by differentiating the following expressions:

$$\begin{aligned} x &= r \cos\varphi \cos(\lambda + \theta g) \\ y &= r \cos\varphi' \sin(\lambda + \theta g) \\ z &= r \sin\theta' \end{aligned} \tag{12}$$

where θg is the Greenwhich hour angle of the true equinox of date. The specific partial derivatives for the 3 x 3 matrix in (11) are as follows (Kahler and Wells, 1966, p. 28):

$$\begin{aligned} \frac{\partial r}{\partial x} &= \frac{x}{r} & \frac{\partial \varphi'}{\partial x} &= \frac{-xz}{r^2\sqrt{x^2+y^2}} & \frac{\partial \lambda}{\partial x} &= \frac{-y}{x^2+y^2} \\ \frac{\partial r}{\partial y} &= \frac{y}{r} & \frac{\partial \varphi'}{\partial y} &= \frac{-yz}{r^2\sqrt{x^2+y^2}} & \frac{\partial \lambda}{\partial y} &= \frac{x}{x^2+y^2} \\ \frac{\partial r}{\partial z} &= \frac{z}{r} & \frac{\partial \varphi'}{\partial z} &= \frac{\sqrt{x^2+y^2}}{r^2} & \frac{\partial \lambda}{\lambda z} &= 0 \end{aligned} \tag{13}$$

The derivatives of V, in (11), can be formed as the sum of the derivatives of the components (i.e. U and T) of V. We have for the derivatives of U:

$$\frac{\partial U}{\partial r} = \frac{-kM}{r^2}\left[1+\sum_{\ell=2}^{\infty}\left(\frac{a}{r}\right)^{\ell}(\ell+1)\sum_{m=0}^{\ell}(\overline{C}_{\ell m}\cos m\lambda \right. \tag{14}$$

$$\left. +\overline{S}_{\ell m}\sin m\lambda)\,\overline{P}_{\ell m}(\sin\varphi')\right]$$

$$\frac{\partial U}{\partial \varphi'} = \frac{kM}{r}\sum_{\ell=2}^{\infty}\left(\frac{a}{r}\right)^{\ell}\sum_{m=0}^{\ell}(\overline{C}_{\ell m}\cos m\lambda+\overline{S}_{\ell m}\sin m\lambda)\,\frac{d\overline{P}_{\ell m}(\sin\varphi')}{d\varphi'} \tag{15}$$

$$\frac{\partial U}{\partial \lambda} = \frac{-kM}{r}\sum_{\ell=2}^{\infty}\left(\frac{a}{r}\right)^{\ell}\sum_{m=0}^{\ell} m(\overline{C}_{\ell m}\sin m\lambda-\overline{S}_{\ell m}\cos m\lambda)\overline{P}_{\ell m}(\sin\varphi') \tag{16}$$

For the derivatives of T we write (Heiskanen and Moritz, 1967, p. 234):

$$\frac{\partial T}{\partial r} = \frac{R}{4\pi}\iint_{\sigma}\Delta g'\,\frac{\partial S}{\partial r}\,d\sigma \tag{17}$$

$$\frac{\partial T}{\partial \varphi'} = \frac{-R}{4\pi}\iint_{\sigma}\Delta g'\,\frac{\partial S}{\partial \psi}\cos\alpha\,d\sigma \tag{18}$$

$$\frac{\partial T}{\partial \lambda} = \frac{-R\cos\varphi'}{4\pi}\iint_{\sigma}\Delta g'\,\frac{\partial S}{\partial \varphi}\sin\alpha\,d\sigma \tag{19}$$

where α is the azimuth from the satellite subpoint to the gravity anomaly block $d\sigma$. We have (ibid, p. 235):

$$S(r,\psi) = t\left[\frac{2}{D}+1-3D-t\cos\psi(5+3\ln\frac{1-t\cos\psi+D}{2})\right] \tag{20}$$

$$\frac{\partial S(r,\psi)}{\partial r} = \frac{-t^2}{R}\left[\frac{1-t^2}{D^3} + \frac{4}{D} + 1 - 6D - t\cos\psi\,(13 + 6\ln\frac{1-t\cos\psi+D}{2})\right] \tag{21}$$

$$\frac{\partial S(r,\psi)}{\partial \psi} = -t^2\sin\psi\left[\frac{2}{D^3} + \frac{6}{D} - 8 - 3\,\frac{1-t\cos\psi-D}{D\sin^2\psi} - 3\ln\frac{1-t\cos\psi+D}{2}\right] \tag{22}$$

where:

$$t = \frac{R}{r} \text{ and } D = (1 - 2t\cos\psi + t^2)^{\frac{1}{2}} \tag{23}$$

The values of ψ and α may be computed from the following equations valid for a sphere.

$$\cos\psi = \sin\varphi'\sin\varphi_g' + \cos\varphi'\cos\varphi_g'\cos(\lambda_g - \lambda) \tag{24}$$

$$\sin\alpha = \frac{\cos\varphi_g'\sin(\lambda_g-\lambda)}{\sin\varphi'} \tag{25}$$

$$\cos\alpha = \frac{\cos\varphi'\sin\varphi_g - \sin\varphi'\cos\varphi_g'\cos(\lambda_g-\lambda)}{\sin\varphi'} \tag{26}$$

where:

φ', λ' are the geocentric latitude and longitude of the satellite subpoint;
φ_g', λ_g' are the geocentric latitude and longitude of the gravity anomaly block.

In practice the summations on ℓ in the potential coefficient equations are carried to some ℓmax instead of infinity. In addition, the integration dealing with gravity anomalies are carried out by a numerical integration over discrete blocks placed on the surface of the sphere approximating the earth.

The precise implementation of the numerical integration technique is somewhat complicated due to the desire to keep the numerical integration errors within bounds. Some of the problems involved are described by Hajela (1972) and will be discussed in this report briefly in the following section.

If one were interested in orbit generation only, considering the gravitational field of the earth represented by potential coefficients and residual gravity anomalies, it would only be necessary to implement the equations of this section in an orbit generation program. However, our goal is more general than this in that we wish to estimate the residual anomalies from the satellite observations and not necessarily just to incorporate anomalies in orbit generation procedures.

4. The Anomaly Variational Equations

In developing the observation equations (equation 6) it is necessary to determine the partial derivatives of the observations with respect to the parameters being estimated. In the type of solutions being described in this method, the partial derivatives needed are computed through the numerical integration of the variational equations. A discussion of the principles of this problem may be found in a paper by Riley et als (1967) or in Conte (1962).

From equation (6) we see that it is necessary, in the general orbit estimation problem, to determine the following derivatives:

$$\frac{\partial \underline{\underline{r}}}{\partial \underline{r}_o} \quad \frac{\partial \underline{\underline{r}}}{\partial \underline{\dot{r}}_o} \quad \frac{\partial \underline{\underline{r}}}{\partial \underline{\Delta g'}} \quad \frac{\partial \underline{\underline{r}}}{\partial \underline{p}} \quad \frac{\partial \underline{\underline{r}}}{\partial \underline{X}}$$

If we were also estimating potential coefficients the derivative of $\underline{r}$ with respect to those coefficients would be added to this test.

We now let β_k be any one of the individual parameters to be estimated. For example, β_k may be a single gravity anomaly. Then the variational equations with respect to β_k may be written in general as:

$$\frac{\partial \ddot{x}}{\partial \beta_k} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \beta_k} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \beta_k} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \beta_k} + \frac{\partial f}{\partial \beta_k} \tag{27}$$

$$\frac{\partial \ddot{y}}{\partial \beta_k} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial \beta_k} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial \beta_k} + \frac{\partial g}{\partial z}\frac{\partial z}{\partial \beta_k} + \frac{\partial g}{\partial \beta_k} \tag{28}$$

$$\frac{\partial \ddot{z}}{\partial \beta_k} = \frac{\partial h}{\partial x}\frac{\partial x}{\partial \beta_k} + \frac{\partial h}{\partial y}\frac{\partial y}{\partial \beta_k} + \frac{\partial h}{\partial z}\frac{\partial z}{\partial \beta_k} + \frac{\partial h}{\partial \beta_k} \tag{29}$$

where: $f=\ddot{x}$, $g=\ddot{y}$, $h=\ddot{z}$. In practice f, g and h are considered to be due to the gravitational field implied by the initial or reference set of potential coefficients.

We now define the following terms which will allow us to express (27), (28) and (29) more compactly:

$$\xi = \frac{\partial x}{\partial \beta_k}, \quad \eta = \frac{\partial y}{\partial \beta_k}, \quad \zeta = \frac{\partial z}{\partial \beta_k}$$

$$\ddot{\xi} = \frac{\partial \ddot{x}}{\partial \beta_k}, \quad \ddot{\eta} = \frac{\partial \ddot{y}}{\partial \beta_k}, \quad \ddot{\zeta} = \frac{\partial \ddot{z}}{\partial \beta_k}$$

$$c_x = \frac{\partial f}{\partial \beta_k}, \quad c_y = \frac{\partial g}{\partial \beta_k}, \quad c_z = \frac{\partial h}{\partial \beta_k}$$

$$B = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial z} \end{bmatrix}$$

Then we can write:

$$\begin{bmatrix} \ddot{\xi} \\ \ddot{\eta} \\ \ddot{\zeta} \end{bmatrix} = B \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} \tag{30}$$

The necessary expressions for the evaluation of the B matrix are given in several sources, for example, Haverland (1971). The evaluation of the needed partials, that is, ξ, η, ζ, is carried out by the numerical integration of (30). This integration can be done at the same time as the orbit integration is being carried out.

For this report we are primarily interested in the evaluation of the position derivatives with respect to the anomalies. To do this we must evaluate the c_x, c_y, c_z values when β_k is a gravity anomaly. To do this we can write.

$$f = \frac{\partial U}{\partial x} + \frac{\partial T}{\partial x} \tag{31}$$

$$g = \frac{\partial U}{\partial y} + \frac{\partial T}{\partial y} \tag{32}$$

$$h = \frac{\partial U}{\partial z} + \frac{\partial T}{\partial z} \tag{33}$$

Then for the ith anomaly we would have:

$$c_{x_i} = \frac{\partial}{\partial \Delta g_i} \left(\frac{\partial T}{\partial x} \right) \tag{34}$$

$$c_{y_i} = \frac{\partial}{\partial \Delta g_i} \left(\frac{\partial T}{\partial y} \right) \tag{35}$$

$$c_{z_i} = \frac{\partial}{\partial \Delta g_i} \left(\frac{\partial T}{\partial z} \right) \tag{36}$$

To determine the derivatives of T with respect to x, y, z we write:

$$\begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial x} & \frac{\partial \varphi'}{\partial x} & \frac{\partial \lambda}{\partial x} \\ \frac{\partial r}{\partial y} & \frac{\partial \varphi'}{\partial y} & \frac{\partial \lambda}{\partial y} \\ \frac{\partial r}{\partial z} & \frac{\partial \varphi'}{\partial z} & \frac{\partial \lambda}{\partial z} \end{bmatrix} \begin{bmatrix} \frac{\partial T}{\partial r} \\ \frac{\partial T}{\partial \varphi'} \\ \frac{\partial T}{\partial \lambda} \end{bmatrix} \tag{37}$$

We can then differentiate the derivatives of T as given by equations (17), ((18) and (19)) to write for c_x, c_y, c_z for the ith anomaly:

$$
\begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}_i = \begin{bmatrix} \frac{\partial r}{\partial x} & \frac{\partial \varphi'}{\partial x} & \frac{\partial \lambda}{\partial x} \\ \frac{\partial r}{\partial y} & \frac{\partial \varphi'}{\partial y} & \frac{\partial \lambda}{\partial y} \\ \frac{\partial r}{\partial z} & \frac{\partial \varphi'}{\partial z} & \frac{\partial \lambda}{\partial z} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}_i \tag{38}
$$

where:

$$
A_i = \frac{R}{4\pi} \frac{\partial S(r,\psi)}{\partial r} d\sigma \tag{39}
$$

$$
B_i = \frac{-R}{4\pi} \frac{\partial S(r,\psi)}{\partial \psi} \cos\alpha \, d\sigma \tag{40}
$$

$$
C_i = \frac{-R\cos\varphi'}{4\pi} \frac{\partial S(r,\psi)}{\partial \psi} \sin\alpha d\sigma \tag{41}
$$

Thus we need to evaluate, for the variational equations, equations (21), (22), (24), (25), and (26), for each of the anomalies that are considered as unknowns in the solutions.

The numerical evaluation of the A, B, and C coefficients requires careful consideration when dealing with anomaly blocks of fairly large size. If we were dealing with very small blocks, then the computation of distances and azimuths from the subsatellite point to the center of the anomaly block would be of sufficient accuracy. However, in dealing with 15° equal area blocks (as in this report), the numerical integration over the anomaly block must be considered. The actual analysis of this problem has been given by Hajela (1972). He recommended a procedure that would limit the numerical integration error as well as would minimize the computer time needed for the evaluation of the quantities needed for A, B and C. In his procedure a given anomaly block is divided into a specified number of sub-blocks. For each sub-block a c_x, c_y, c_z value was computed. A mean value was then formed for the large block from the individual sub-block values. The number of sub-blocks used was set as a function of the spherical distance from the sub-satellite point to the anomaly blocks. Specifically the following sub-block division was used when generating the values of c_x, c_y, c_z needed for the variational equation integration:

$0 < \psi < 12^\circ\!.5$,	an	anomaly	block	was	divided	into	16	sub-blocks
$12^\circ \le \psi < 20^\circ$,	"	"	"	"	"	"	9	"
$20^\circ \le \psi < 35^\circ$,	"	"	"	"	"	"	4	"
$35^\circ \le \psi \le 180^\circ$,	"	"	"	"	"	"	1	"

Finally we should note that the evaluation of the c_x, c_y, c_z values for each anomaly unknown must be done for each integration step in the variational operation integration procedure.

5. Anomaly Constraints.

In the standard gravitational field estimation using potential coefficients certain potential coefficients are usually forced to be zero by excluding them from the coefficients being solved for. Specifically in order to assure the coordinate system has its origin at the center of mass of the earth $\overline{C}_{1,0}$, $\overline{C}_{1,1}$ and $\overline{S}_{1,1}$ are forced to be zero. In addition, if the z axis of the coordinate system is to be referenced to the mean rotation axis, the $\overline{C}_{2,1}$ and $\overline{S}_{2,1}$ coefficients should be forced to be zero.

In the solutions method described in this paper an alternate procedure must be used for imposing the needed conditions. To do this we first consider a spherical harmonic representation of the gravity anomalies on the earth in the following form:

$$\Delta g = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \left(a_{\ell m} \cos m\lambda + b_{\ell m} \sin m\lambda \right) P_{\ell m}(\sin\varphi') \tag{42}$$

In practice the summation to ∞ is replaced by a summation to an ℓmax that will depend on the size of the anomaly block being represented. The coefficients in equation (42) can be determined from the following:

$$\left\{ \begin{matrix} a_{\ell m} \\ b_{\ell m} \end{matrix} \right\} = \frac{1}{4\pi} \iint_{\sigma} \Delta g \left\{ \begin{matrix} \cos m\lambda \\ \sin m\lambda \end{matrix} \right\} P_{\ell m}(\sin\varphi') \tag{43}$$

In both (42) and (43) we assume that the anomalies refer to an ellipsoidal reference system. If the anomalies were referred to a higher order reference surface (i.e. the anomalies were Δg values) the coefficients found in (43) would be referred to the reference surface to which the $\Delta g'$ values were referred.

In order to assure that the origin of our coordinate system is at the center of mass of the earth the $\overline{a}_{1,0}$, $\overline{a}_{1,1}$ and $\overline{b}_{1,1}$ coefficients implied by the adjusted anomalies must be zero. For the z axis of the coordinate system to coincide with the mean rotation axis the $\bar{a}_{2,1}$ and the $\bar{b}_{2,1}$ coefficients must be zero.

Evaluation of (43) for $\ell = m = 0$ yields the mean anomaly, Δg_0, over the earth:

$$\Delta g_0 = a_{0,0} = \frac{1}{4\pi} \iint_{\sigma} \Delta g \, d\sigma \tag{44}$$

In the estimation of the anomalies in the adjustment with the satellite data, (and perhaps terrestrial gravity data), a value for Δg_0 should be enforced on the solution so that it is zero or some value computed on the basis of knowledge of the parameters of a mean earth ellipsoid. For example assume that we are given anomalies with respect to a gravity formula that has an equatorial gravity, γ_e, that differs from the best estimate ($\bar{\gamma}_e$) of equatorial gravity. Then we can find Δg_0 from (Heiskanen and Moritz, 1967, p. 106):

$$\Delta g_0 = \bar{\gamma}_e - \gamma_e \tag{45}$$

Using the above information we can now go back and write equation (8) more explicitly for the six condition equations involved. We have:

$$G_1 \left(\frac{1}{4\pi} \iint_{\sigma} \Delta g d\sigma - \Delta g_0 \right) = 0 \tag{46}$$

$$G_2 \left(\frac{1}{4\pi} \iint_{\sigma} \Delta g P_{1,0} \, d\sigma \right) = 0 \tag{47}$$

$$G_3 \left(\frac{1}{4\pi} \iint_{\sigma} \Delta g P_{1,1} \cos\lambda \, d\sigma \right) = 0 \tag{48}$$

$$G_4 \left(\frac{1}{4\pi} \iint_{\sigma} \Delta g P_{1,1} \sin\lambda \, d\sigma \right) = 0 \tag{49}$$

$$G_5 \left(\frac{1}{4\pi} \iint_{\sigma} \Delta g P_{2,1} \cos\lambda \, d\sigma \right) = 0 \tag{50}$$

$$G_6 \left(\frac{1}{4\pi} \iint_{\sigma} \Delta g P_{2,1} \sin\lambda \, d\sigma \right) = 0 \tag{51}$$

The misclosures, W_G are equal to the values of equations (46) through (51) evaluated with the approximate values of the anomalies. (Note that in the implementation of these procedures for this study all approximate values of the anomalies were set to zero.)

The coefficients in the B_{GX} matrix (see equation (9)) are simply the coefficients of the anomalies as they appear in equations (46) through (51). For example, for each anomaly, equation (46) implies a coefficient such as $d\sigma_i/4\pi$ for the ith anomaly. From (48) the coefficient for the ith anomaly is:

$$c_i = \frac{1}{4\pi} P_{1,1} (\sin\varphi_i) \cos\lambda_i d\sigma_i \tag{52}$$

if the blocks are sufficiently small. In practice we formed integrated mean values of the coefficients where the integration was carried out over the anomaly block. Thus, for example, the exact coefficient used for the condition given by (48) is found by forming the integrated mean value of (48). We have, for a block defined by latitude limits φ_1 and φ_2 and longitude limits λ_1 and λ_2:

$$\bar{c}_i = \frac{1}{4\pi} \int_{\varphi_1}^{\varphi_2} \int_{\lambda_1}^{\lambda_2} \cos\varphi \cos\lambda \cos\varphi d\varphi d\lambda \tag{53}$$

$$\bar{c}_i = \frac{\sin\lambda_2 - \sin\lambda_1}{8\pi} \left[\varphi_2 - \varphi_1 + \frac{\sin 2\varphi_2 - \sin 2\varphi_1}{2} \right] \tag{54}$$

We next summarize the integrated anomaly coefficient for each of the conditions represented by equations (46) through (51)

For equation (46):

$$\bar{c}_i = \frac{(\sin\varphi_2 - \sin\varphi_1)(\lambda_2 - \lambda_1)}{4\pi} \tag{55}$$

For equation (47):

$$\bar{c}_i = \frac{-(\cos^2\varphi_2 - \cos^2\varphi_1)(\lambda_2 - \lambda_1)}{8\pi} \tag{56}$$

For equation (48):

$$\bar{c}_i = \frac{(\sin\lambda_2 - \sin\lambda_1)}{8\pi} (\varphi_2 - \varphi_1 + \frac{\sin 2\varphi_2 - \sin 2\varphi_1}{2}) \tag{57}$$

For equation (49):

$$\bar{c}_i = -(\cos\lambda_2 - \cos\lambda_1)(\varphi_2 - \varphi_1 + \frac{\sin 2\varphi_2 - \sin 2\varphi_1}{2}) \tag{58}$$

For equation (50): $$\bar{c}_1 = \frac{-(\sin\lambda_2 - \sin\lambda_1)}{4\pi}(\cos^3\varphi_2 - \cos^3\varphi_1) \tag{59}$$

For equation (51): $$\bar{c}_1 = \frac{(\cos\lambda_2 - \cos\lambda_1)}{4\pi}(\cos^3\varphi_2 - \cos^3\varphi_1) \tag{60}$$

6. Planned Analysis.

In order to carry out a test of the direct determination of anomalies from satellite data we intend to analyze optical satellite data for a number of arcs of time duration of about 5 to 7 days. For each arc we will solve for arc dependent quantities (such as epoch position and velocity vectors, air drag parameters, etc.), as well as the coordinates of selected observation stations and 184, 15° equal area residual anomalies. These residual anomalies can be converted back to anomalies Δg, referred to an ellipsoidal gravity formula, by adding to the residual anomalies, the anomalies Δg_{PC} implied by the reference set of potential coefficients used in the orbit generation. We have:

$$\Delta g = \Delta g_{PC} + \Delta g' \tag{61}$$

where

$$\Delta g_{PC} = \gamma \sum_{\ell=2}^{\ell_{max}} (\ell-1) \sum_{m=0}^{\ell} (\bar{C}^*_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda)\bar{P}_{\ell m}(\sin\varphi') \tag{62}$$

where $\bar{C}^*_{\ell m}$ are the $\bar{C}_{\ell m}$ values referred to values implied by a reference ellipsoid of a specific flattening and $\gamma = 979.8$ mgals.

We can also determine the potential coefficients implied by the new solution by writing:

$$\begin{Bmatrix} \bar{C}_{\ell m} \\ \bar{S}_{\ell m} \end{Bmatrix}_n = \begin{Bmatrix} \bar{C}_{\ell m} \\ \bar{S}_{\ell m} \end{Bmatrix}_r + \begin{Bmatrix} \bar{C}'_{\ell m} \\ \bar{S}'_{\ell m} \end{Bmatrix} \tag{63}$$

where the subscript n denotes the coefficients of the solution; r designates the reference set of coefficients used in the orbit generation and the primed coefficients used in the orbit generation and the primed coefficients are computed from the adjusted residual anomalies using:

$$\begin{Bmatrix} \overline{C}'_{\ell m} \\ \overline{S}'_{\ell m} \end{Bmatrix} = \frac{1}{4\pi\gamma(\ell-1)} \iint_{\sigma} \Delta g' \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \overline{P}_{\ell m} (\sin\varphi')\, d\sigma \tag{64}$$

with the integration carried out by numerical integration over the global set of adjusted anomalies found from the solution.

The anomalies computed from (61) with the $\Delta g'$ values found from a satellite solution can be compared with existing terrestrial gravity material. In addition the potential coefficients computed from (63) can be compared to potential coefficient estimates determined from conventional techniques.

7. The Orbit Determination and Geodetic Recovery Program.

In order to process our satellite observations to determine station coordinates and the unknown gravity anomalies (as well as other quantities that are dependent on the arc of the satellite that is being processed) we need an accurate orbit determination program that can be used for the estimation of the parameters of interest. One such program is the Geodyn program that was developed by the Wolf Research and Development Corporation. This program contains almost all the sophisticated features that are needed in the accurate estimation of quantities of geodetic interest from the processing of many types of satellite observations. In this program careful attention has been given to numerical integration techniques, both in the orbit integration as well as in the integration of the variation equations. In addition such small, but important effects, as air drag, radiation pressure, earth tides, polar motion, time corrections, have been considered. A description of the data input for the version of Geodyn that was made available to us (which was received February 28, 1972) may be found in Martin (1972). A detailed development of the theory implemented in Geodyn may be found in a set of reports produced by Wolf (see the list of references for details).

After the receipt of the February 1972 version of Geodyn, it was modified to incorporate the procedures needed to estimate gravity anomalies directly from the analysis of satellite data and in combination with terrestrial mean gravity anomalies. This required the coding of the equations given in the previous section and the incorporation of such coding (either as replacement coding or new coding) in Geodyn. In addition to these changes, procedures were also worked out in that we could process arcs with the accumulation of the normal equations with a total solution (i.e. an outer iteration) either as a satellite solution or a combination solution being carried out at any time after an arbitrary number of arc normals had been accumulated. This allowed us to accumulate the normals for (say) n arcs and then make a satellite alone solution after which we could make a combination solution.

A discussion of the new input cards needed for the modified Geodyn program may be found in an internal report by Karki (1973). In addition Karki gives sample input deck sets for the modified Geodyn as well as other pertinent information.

8. Data To Be Used.

8.1 Satellite data and preprocessing.

We decided to use only optical satellite data for the test of the method described in this report. We initially received data from 23 satellites. From this data we selected data from 10 satellites in 79 arcs of approximate 7 day duration. These satellites and arcs were selected to obtain a good inclination distribution as well as obtaining arcs with sufficiently dense data.

Of the 79 initial arcs considered, 39 were processed in an "inner iteration" cycle to obtain converged starting elements. This inner iteration was carried out using the Geodyn program starting from initial elements and other starting values estimated by Nickerson (1972). We give in Table 1 a summary of the 39 arcs used at some time in this study. To obtain the root mean square orbit fit in seconds multiply the RMS fit by 2". We give in Table A of the appendix the converged epoch position and velocity vectors and other information for the 39 arcs considered.

Although several solutions with a different number of arcs were run, the two main solutions were a 29 arc and a 39 arc solution. A summary of the data used in each of these solutions, by satellite, is given in Tables 2 and 3.

The potential coefficients (basically those of the SAO Standard Earth (I)) used for the initial orbit determination were complete to degree 8 with additional coefficients to degree 21. The complete list of these coefficients, which form the reference potential, is given in Table 4.

The station coordinates used in the initial fitting were a set of values updated from those in the version of Geodyn we were working with. The values of the coordinates (referred to a reference ellipsoid with an equatorial radius equal to 6378155 meters and an inverse flattening of 1/298.255) are given in Table 5.

Table 1

Information Related to Arcs
Used in Solution

ARC NO.	SAT. NAME	EPOCH MM	DD	YY	LENGTH DAYS	ACC. OBS.	RMS FIT
1	ANNA	1	2	66	5.0	276	1.693
2	BEB	2	26	67	6.0	188	1.872
3	BEC	4	4	67	5.5	348	1.847
4	COURIER	12	31	66	7.0	457	1.593
5	DIC	3	17	67	7.0	216	2.236
6	GEOS A	2	16	66	7.0	1173	1.095
7	GEOS B	4	14	68	7.0	1657	2.243
8	OSCAR	4	8	66	6.5	537	2.195
9	OVI-2	11	11	66	7.0	281	2.102
10	ANNA	12	22	65	5.0	256	1.482
11	BEB	3	16	67	6.0	146	2.422
12	BEC	3	25	66	5.5	381	1.220
13	COURIER	7	7	67	7.0	296	1.450
14	DIC	2	24	67	7.0	214	1.777
15	DID-7	5	28	67	7.0	590	2.122
16	GEOS A	12	31	65	7.0	1055	1.907
17	GEOS B	10	6	68	7.0	1485	1.497
18	OSCAR	4	15	66	6.5	474	2.500
19	OVI-2	11	4	66	7.0	288	2.288
20	ANNA	12	11	65	5.0	154	1.301
21	BEC	4	23	66	5.5	348	1.654
22	COURIER	1	8	67	7.0	375	1.577
23	DID-7	5	14	67	7.0	1611	1.506
24	GEOS A	11	15	66	7.0	987	1.527
25	GEOS B	9	15	68	7.0	2655	1.358
26	OSCAR	4	1	66	7.0	433	1.974
27	OVI-2	11	18	66	7.0	196	2.153
28	BEC	3	14	66	5.5	284	1.114
29	COURIER	1	27	67	7.0	290	1.761
30	DID-7	5	7	67	7.0	1365	1.525
31	GEOS A	7	9	66	7.5	3468	1.135
32	GEOS B	6	8	68	6.5	2172	2.013
33	OSCAR	4	22	66	7.0	329	2.797
34	BEC	3	17	67	5.5	268	2.467
35	COURIER	7	14	67	7.0	284	1.423
36	DID-7	3	5	67	7.0	435	1.818
37	GEOS A	9	25	66	7.5	3190	1.275
38	BEC	4	15	67	5.0	242	1.360
39	COURIER	6	23	67	6.0	256	1.176

Table 2

Arc Data for the 29 Arc Solution

SAT ID	SAT. NAME	INC.	ECC.	APO.H. KM	PER.H. KM	NO.OF ARCS	TOT. OBS.
620601	ANNA	50	0.007	1190	1080	3	686
640641	BEB	80	0.014	1099	898	2	334
650321	BEC	41	0.025	1324	947	4	1361
600131	COURIER	28	0.017	1220	971	4	1418
670111	DIC	40	0.053	1355	578	2	430
670141	DID-7	39	0.084	1885	600	2	2201
650891	GEOS A	59	0.072	2277	1120	3	3215
680021	GEOS B	106	0.033	1591	1083	3	5797
660051	OSCAR	90	0.023	1210	861	3	1444
650781	OVI-2	144	0.182	3445	421	3	765

Table 3

Arc Data for the 39 Arc Solution

SAT ID	SAT. NAME	INC.	ECC.	APO.H. KM	PER.H. KM	NO.OF ARCS	TOT. OBS.
620601	ANNA	50	0.007	1190	1080	3	686
640641	BEB	80	0.014	1099	898	2	334
650321	BEC	41	0.025	1324	947	6	1871
600131	COURIER	28	0.017	1220	971	6	1958
670111	DIC	40	0.053	1355	578	2	430
670141	DID-7	39	0.084	1885	600	4	4001
650891	GEOS A	59	0.072	2277	1120	5	9873
680021	GEOS B	106	0.033	1591	1083	4	7969
660051	OSCAR	90	0.023	1210	861	4	1773
650781	OVI-2	144	0.182	3445	421	3	765

Table 4

Initial or Reference Set of Potential Coefficients

L	M	C(L,M)X10^{6}	S(L,M)X10^{6}
2	0	-484.167	
2	2	2.380	-1.351
3	0	0.959	
3	1	1.936	0.266
3	2	0.735	-0.539
3	3	0.561	1.621
4	0	0.531	
4	1	-0.572	-0.469
4	2	0.330	0.662
4	3	0.852	-0.191
4	4	-0.053	0.230
5	0	0.069	
5	1	-0.079	-0.103
5	2	0.630	-0.232
5	3	-0.521	0.007
5	4	-0.265	0.064
5	5	0.156	-0.593
6	0	-0.139	
6	1	-0.047	-0.027
6	2	0.069	-0.366
6	3	-0.054	0.031
6	4	-0.044	-0.518
6	5	-0.313	-0.458
6	6	-0.040	-0.155
7	0	0.093	
7	1	0.197	0.156
7	2	0.364	0.163
7	3	0.250	0.018
7	4	-0.152	-0.102
7	5	0.076	0.054
7	6	-0.209	0.063
7	7	0.055	0.097
8	0	0.029	
8	1	-0.076	0.065
8	2	0.026	0.039
8	3	-0.037	0.004
8	4	-0.212	-0.012
8	5	-0.053	0.118
8	6	-0.017	0.318
8	7	-0.009	0.031
8	8	-0.248	0.102
9	0	0.023	
9	1	0.117	0.012
9	2	-0.004	0.035
9	9	0.185	0.210
10	0	0.077	
10	1	0.105	-0.126
10	2	-0.105	-0.042
10	3	-0.065	0.030
10	4	-0.074	-0.111
10	9	0.104	-0.064
11	0	-0.042	
11	1	-0.053	0.015

L	M	$C(L,M)\times10^6$	$S(L,M)\times10^6$
11	11	0.027	0.056
12	0	0.008	
12	1	-0.163	-0.071
12	2	-0.103	-0.005
12	11	-0.054	-0.311
12	12	-0.033	-0.005
13	0	0.024	
13	12	-0.070	0.075
13	13	-0.055	0.124
14	0	0.014	
14	1	-0.015	0.005
14	11	0.000	-0.000
14	12	0.003	-0.028
14	13	0.023	0.055
14	14	-0.046	-0.025
15	0	0.031	
15	9	-0.001	-0.002
15	12	-0.076	-0.001
15	13	-0.022	0.031
15	14	0.002	-0.022
16	0	-0.033	
16	14	-0.017	0.001
17	0	-0.014	
17	13	0.036	0.049
17	14	-0.014	-0.002
18	0	0.038	
19	0	0.035	
20	0	0.001	
21	0	-0.022	

Table 5

Initial Station Coordinates

STATION NUMBER	LATITUDE			LONGITUDE			HEIGHT (M)
1021	38	25	49.79	282	54	48.61	-54.00
1022	26	32	53.14	278	8	4.16	-42.00
1024	-31	23	25.88	136	52	15.14	130.00
1028	-33	8	58.88	289	19	53.66	710.00
1030	35	19	47.89	243	5	58.92	876.00
1031	-25	53	1.44	27	42	26.21	1541.00
1032	47	44	29.27	307	16	46.14	48.00
1034	48	1	21.53	262	59	19.51	203.00
1035	51	26	46.40	359	18	7.93	90.00
1036	64	58	36.75	212	28	30.52	283.00
1037	35	12	7.28	277	7	41.16	850.00
1038	-35	37	32.68	148	57	14.85	950.00
1042	35	12	7.30	277	7	40.86	850.00
1043	-19	0	32.59	47	17	59.29	1360.00
4732	37	52	1.99	284	32	57.68	-54.07
4733	37	52	2.00	284	32	57.66	-54.07
4734	37	20	49.83	284	5	48.13	-60.47
7034	48	1	21.53	262	59	19.51	203.00
7036	26	22	46.52	261	40	7.25	8.00
7037	38	53	36.24	267	47	40.87	213.00
7039	32	21	49.93	295	20	35.41	-27.00
7040	18	15	28.58	294	0	23.53	-18.00
7043	39	1	15.15	283	10	20.43	-6.00
7045	39	38	48.14	255	23	38.47	1745.00
7071	27	1	13.76	279	53	12.55	-37.68
7072	27	1	14.16	279	53	12.73	-37.00
7073	27	1	14.10	279	53	12.96	-38.17
7074	27	1	14.32	279	53	13.00	-37.52
7075	46	27	21.53	279	3	10.41	221.00
7076	18	4	34.46	283	11	27.13	405.00
7077	38	59	57.00	283	9	37.71	-6.00
7078	37	51	46.96	284	29	27.63	-55.00
7079	-24	54	23.40	113	43	15.59	-14.00
8010	46	52	37.18	7	27	53.35	933.22
8015	43	55	57.55	5	42	44.74	694.32
8019	43	43	33.05	7	17	58.68	405.22
8030	48	48	22.64	2	13	45.94	190.01
9001	32	25	25.05	253	26	49.07	1631.44
9002	-25	57	35.95	28	14	52.84	1568.57
9004	36	27	46.75	353	47	37.14	71.95
9005	35	40	23.01	139	32	16.65	96.06
9006	29	21	34.72	79	27	27.60	1884.68
9007	-16	27	56.74	288	30	24.82	2491.58
9008	29	38	13.88	52	31	11.53	1593.29
9009	12	5	25.20	291	9	44.72	-13.84
9010	27	1	14.15	279	53	13.56	-11.88
9011	-31	56	34.68	294	53	36.93	633.91
9012	20	42	26.16	203	44	33.98	3056.17
9021	31	41	2.95	249	7	18.36	2339.00
9023	-31	23	25.82	136	52	43.96	143.49
9025	36	0	19.92	139	11	31.17	879.00
9028	8	44	50.71	38	57	32.98	1901.00

9029	-5	55	40.18	324	50	7.39	25.39
9031	-45	53	12.61	292	23	9.40	203.00
9049	27	1	13.72	279	53	12.88	-39.00
9050	42	30	20.94	288	26	30.01	131.00
9091	38	4	44.39	23	55	58.43	490.00
9424	54	44	33.65	249	57	22.12	654.00
9425	34	57	50.56	242	5	7.75	729.00
9426	60	12	39.50	10	45	2.69	595.00
9427	16	44	38.47	190	29	8.75	-7.00
9435	60	9	42.31	24	57	5.41	40.00

Both the potential coefficients and the station coordinates were basically those used as starting values for the GEM1 solution (Lerch et als., 1972).

8.2 Terrestrial Gravity Data

In carrying out a combination solution it is necessary to have estimates of the terrestrial anomalies and their accuracy for the block subdivision of the study. Here we elected to use 184 15° near equal area blocks. This size was selected as an optimum choice between too large a subdivision and too many blocks. Future analysis could use smaller blocks such as 10° equal area blocks.

The blocks were chosen to have a 15° latitude extent with the longitude extent chosen as some integer degree that wouldyield a near equal area block. The 15° anomalies, in areas where there was some known 1°x 1° anomalies, were estimated by least squares prediction techniques. In empty areas model anomalies (based on topographic isostatic information) were used. Of the 184 values only 10 were estimated on the basis of no actual gravity data while a total of 23355 1°x 1° anomalies were considered in the estimation procedure that used the actual gravity data. All anomalies were estimated with respect to the following normal gravity formula:

$$\gamma = \gamma_e (1+0.00530243 \sin^2\varphi - 0.00000587 \sin^2 2\varphi) \tag{65}$$

with γ_e equal to 978033.51 mgals.

The accuracy of the 15° anomalies was also available. For use in this study, the accuracy estimates used were found from the following equation.

$$m_{\Delta g} = \sqrt{m_H^2 + (1.5)^2} \tag{66}$$

where m_H is the standard deviation of the 15° anomaly as given by Hajela (1973) while the 1.5 mgals is included to reflect inaccuracy in our knowledge of equatorial gravity and possible base station errors.

Full details of the estimation process may be found in Hajela (1973). The anomaly block borders, the terrestrial anomaly, and the anomaly standard deviation, as computed from (66) are given in Table 7.

9. Solutions and Results

After the initial arc convergence, one final run (for each arc) was made in an "outer iteration" mode using the modified Geodyn program. At this point the normal equations for the unknowns common to all arcs were formed. These unknowns were the 184 anomaly unknowns and the station coordinates for seven stations, all other stations being held fixed in the adjustment. The seven stations for whom adjusted coordinates were determined were selected as those from which the densest satellite observations were available. More stations were not solved for because of core size limitations on the IBM 370/165 computer available for our use at Ohio State. The stations for which adjusted coordinates were sought were: 9001, 9002, 9004, 9006, 9007, 9012 and 9023.

The normal equations were accumulated for sequential arcs with a satellite alone solution being made after 5, 10, 15, 20, 25, 26, 28, 29, 30 and 39 arcs. In addition a combination solution was made with the data from the 29 arc run. Also combinations of different arcs were made using, for example, arcs that had the best orbit fits. However, these latter runs showed no essential difference from the original arc combinations.

The value of Δg_0 needed for the anomaly constraints was taken as 0.0 mgals reflecting a best estimate equal to that value connected with equation (65).

From each solution the Δg anomalies were computed using equation (61) while the new potential coefficients were computed using (63) with equation (64) being evaluated by numerical integration over the 184 anomaly blocks. The anomalies were compared with the values of the 184 terrestrial anomalies derived by Hajela. The root mean square anomaly difference and the maximum anomaly differences were computed. These quantities are given in Table 6 for some of the solutions made for this paper.

The potential coefficients found for our different solutions were compared to the coefficients of the GEM3 solution by computing (for solutions made to $\ell=12$ maximum) the correlation coefficient r, the average percentage difference ($\overline{\%}$), and the root mean square coefficient difference(s). Such values are shown in Table 6.

Considering this table we see that as arcs up to 29 are added the satellite alone results show increasing agreement with our terrestrial anomaly data and/or the GEM3 potential coefficients. However, the results from the 39 arc solution show less agreement than the 29 arc satellite alone solution. The reason for this is not clear. Although many items were checked for errors in the 39 arc run, none were found. Perhaps with this number of arcs we need a considerable amount of additional observations on well distributed arcs in order to see a positive improvement in our results.

Table 6

Comparison of 15° Anomalies and Potential Coefficients From Various Solutions

	Comparison of 15° anomalies of various solutions to 15° terrestrial data		Potential Coefficient Comparisons to GEM3		
Solution*	RMS diff.	Max diff.	Corr. coeff. r	Per-diff. %	RMS diff $\Delta \times 10^6$
10 arc sat	17.8mgals	52.4mgals	.967	78.1	.097
15 arc sat	16.8 "	50.2 "	.968	75.5	.093
20 arc sat	13.8 "	41.0 "	.981	58.4	.072
25 arc sat	11.6 "	34.1 "	.986	50.1	.062
29 arc sat	11.6 "	35.3 "	.987	48.4	.060
29 arc comb	6.2 "	23.0 "	.989	43.7	.055
39 arc sat	13.7 "	49.5 "	.983	56.0	.068

*comparisons made to 12,12.

Considering the 29 arc satellite solution as the best of those tried for this report, we proceeded to make a 29 arc combination solution. To do this we first needed to develop a proper scaling factor s. This was done by computing the difference between the anomalies found from the 29 arc satellite solution and the terrestrial data. It was concluded from this analysis that realistic standard deviations from the 29 arc solution would be obtained by multiplying the results from the initial solution by 3. Thus, for the combination solution s^2 was taken to be $1/3^2$. Results on the anomaly and potential coefficient comparisons have been shown in Table 6.

In Table 7 we give information related to the 184 15° blocks. In addition to the block sequence number and the coordinates of the block borders we have the terrestrial anomaly and its standard deviation (as computed from (66)), the anomaly from the 29 arc satellite alone solution, and its standard deviation as obtained directly from the solution, and the anomaly and its standard deviation as found from the 29 arc combination solution.

In Figure 1 we show the location of the 15° equal area blocks. In Figure 2 and Figure 3 we show the anomalies and standard deviations of the 15° blocks

Table 7
Information Related to the 184 15° Equal Area Anomalies

Block No	φ_N	φ_S	λ_W	λ_E	Terr. Δg	Terr. m	29 Arc Sat Δg	29 Arc Sat m	29 Arc Comb Δg	29 Arc Comb m
1	-75.00	-90.00	0.0	120.00	3.4	4.8	7.7	1.3	4.8	1.8
2	-75.00	-90.00	120.00	240.00	-18.0	2.9	-5.5	1.3	-0.4	1.7
3	-75.00	-90.00	240.00	360.00	-6.5	3.3	-1.7	1.2	-3.9	1.7
4	-60.00	-75.00	0.0	40.00	6.7	3.8	-8.7	2.4	2.1	2.7
5	-60.00	-75.00	40.00	80.00	14.4	4.0	12.5	2.1	5.9	2.5
6	-60.00	-75.00	80.00	120.00	-0.5	3.3	-21.4	2.3	-9.7	2.3
7	-60.00	-75.00	120.00	160.00	-9.7	3.4	9.9	2.8	-7.4	2.5
8	-60.00	-75.00	160.00	200.00	-9.4	3.8	-11.2	2.8	5.8	2.7
9	-60.00	-75.00	200.00	240.00	-9.0	4.8	20.5	2.8	4.5	2.9
10	-60.00	-75.00	240.00	280.00	-1.9	4.5	-6.0	2.6	-4.1	2.9
11	-60.00	-75.00	280.00	320.00	10.1	4.2	-6.1	2.6	4.8	2.8
12	-60.00	-75.00	320.00	360.00	-6.6	5.9	10.6	2.4	-1.9	3.1
13	-45.00	-60.00	0.0	24.00	-1.9	5.9	-4.9	3.0	-1.0	3.6
14	-45.00	-60.00	24.00	48.00	0.0	5.6	6.3	2.7	0.6	3.4
15	-45.00	-60.00	48.00	72.00	1.3	5.5	-6.0	2.4	-11.2	3.2
16	-45.00	-60.00	72.00	96.00	1.0	5.7	4.1	1.9	13.7	3.0
17	-45.00	-60.00	96.00	120.00	-2.0	5.9	20.8	2.4	8.8	3.3
18	-45.00	-60.00	120.00	144.00	-2.6	5.9	-36.2	2.9	-12.0	3.5
19	-45.00	-60.00	144.00	168.00	0.4	4.4	32.7	3.3	4.6	3.2
20	-45.00	-60.00	168.00	192.00	-5.1	4.5	-24.1	3.4	-5.3	3.2
21	-45.00	-60.00	192.00	216.00	-1.6	5.7	3.4	2.8	-1.3	3.5
22	-45.00	-60.00	216.00	240.00	1.1	5.9	10.3	3.3	7.6	3.8
23	-45.00	-60.00	240.00	264.00	0.2	5.1	-11.1	3.2	-1.8	3.5
24	-45.00	-60.00	264.00	288.00	-1.9	4.6	4.3	2.8	-2.0	3.0
25	-45.00	-60.00	288.00	312.00	-0.2	3.0	7.8	3.1	0.8	2.5
26	-45.00	-60.00	312.00	336.00	3.2	5.1	-8.5	2.9	-0.1	3.2
27	-45.00	-60.00	336.00	360.00	0.4	5.9	0.7	3.0	-1.8	3.7
28	-30.00	-45.00	0.0	19.00	-1.7	4.7	-16.8	1.8	-16.4	2.5
29	-30.00	-45.00	19.00	38.00	14.0	3.3	8.5	1.7	5.4	2.2
30	-30.00	-45.00	38.00	57.00	7.1	4.9	-0.7	1.7	6.7	2.4
31	-30.00	-45.00	57.00	76.00	-1.6	4.6	-10.0	1.6	-10.2	2.4
32	-30.00	-45.00	76.00	95.00	-1.5	4.4	12.2	1.6	8.3	2.3
33	-30.00	-45.00	95.00	114.00	-15.0	4.3	2.3	1.6	3.3	2.3
34	-30.00	-45.00	114.00	133.00	-14.0	4.5	6.3	1.6	-2.8	2.4
35	-30.00	-45.00	133.00	152.00	4.5	3.1	-11.4	1.8	0.8	2.2
36	-30.00	-45.00	152.00	171.00	-4.5	3.9	-11.3	1.8	-8.4	2.3
37	-30.00	-45.00	171.00	189.00	3.2	3.5	5.0	1.9	-0.1	2.4
38	-30.00	-45.00	189.00	208.00	-7.3	5.0	9.6	1.8	-1.1	2.6
39	-30.00	-45.00	208.00	227.00	-0.4	4.9	-0.6	1.8	9.0	2.8
40	-30.00	-45.00	227.00	246.00	0.6	4.6	3.1	1.8	2.3	2.6
41	-30.00	-45.00	246.00	265.00	-4.3	4.4	-8.9	1.9	-6.7	2.5
42	-30.00	-45.00	265.00	284.00	-1.3	4.0	-8.2	1.8	-10.5	2.3
43	-30.00	-45.00	284.00	303.00	13.6	2.2	15.5	2.0	10.3	1.8
44	-30.00	-45.00	303.00	322.00	-1.2	3.5	-12.8	2.0	-1.3	2.3
45	-30.00	-45.00	322.00	341.00	-0.2	5.1	-2.4	1.8	-4.9	2.6
46	-30.00	-45.00	341.00	360.00	2.4	5.0	21.0	1.9	16.5	2.7
47	-15.00	-30.00	0.0	16.00	1.2	4.0	-14.2	1.1	-13.7	2.0
48	-15.00	-30.00	16.00	33.00	10.8	2.7	11.7	1.0	10.9	1.6
49	-15.00	-30.00	33.00	49.00	1.0	2.8	-7.3	1.1	-1.7	1.6
50	-15.00	-30.00	49.00	65.00	6.0	2.8	5.5	1.1	-1.2	1.6
51	-15.00	-30.00	65.00	82.00	9.9	3.4	-8.9	1.0	-3.1	1.7
52	-15.00	-30.00	82.00	98.00	-14.2	3.6	-0.8	1.0	-3.6	1.7
53	-15.00	-30.00	98.00	115.00	-13.0	3.2	-8.0	0.9	-4.6	1.5
54	-15.00	-30.00	115.00	131.00	-1.0	1.9	18.5	1.0	16.4	1.3
55	-15.00	-30.00	131.00	147.00	2.4	1.6	-17.3	1.1	-8.0	1.2
56	-15.00	-30.00	147.00	164.00	7.0	2.5	22.5	1.1	6.0	1.6
57	-15.00	-30.00	164.00	180.00	20.2	3.6	-16.1	1.2	-3.0	2.0
58	-15.00	-30.00	180.00	196.00	-1.6	4.1	16.2	1.2	9.0	2.0
59	-15.00	-30.00	196.00	213.00	6.0	5.5	-15.6	1.0	-10.0	1.9
60	-15.00	-30.00	213.00	229.00	2.1	5.8	10.2	1.2	5.0	2.1

1	-15.00	-30.00	229.00	245.00	1.6	5.5	-12.1	1.1	-4.3	2.0
2	-15.00	-30.00	245.00	262.00	-0.6	5.1	15.7	1.0	5.1	1.9
3	-15.00	-30.00	262.00	278.00	-2.4	5.7	-5.7	1.1	3.2	2.0
4	-15.00	-30.00	278.00	295.00	17.5	3.7	7.9	1.1	5.2	1.8
5	-15.00	-30.00	295.00	311.00	1.0	2.8	-2.0	1.3	-3.1	1.8
6	-15.00	-30.00	311.00	327.00	-13.7	2.9	6.6	1.2	-1.9	1.8
7	-15.00	-30.00	327.00	344.00	-4.8	5.6	-15.1	1.2	-6.6	2.2
8	-15.00	-30.00	344.00	360.00	-1.9	5.3	6.4	1.3	2.6	2.2
9	0.0	-15.00	0.0	15.00	-3.4	4.8	13.0	1.4	2.6	2.2
0	0.0	-15.00	15.00	30.00	-10.1	3.0	-8.2	1.4	-0.9	2.0
1	0.0	-15.00	30.00	45.00	-10.6	2.4	-11.9	1.3	-4.3	1.8
2	0.0	-15.00	45.00	60.00	-10.0	2.7	15.5	1.2	4.6	1.8
3	0.0	-15.00	60.00	75.00	-11.8	3.3	-0.2	1.3	1.1	1.9
4	0.0	-15.00	75.00	90.00	-21.8	3.8	1.5	1.1	-0.9	1.9
5	0.0	-15.00	90.00	105.00	-6.1	2.8	7.2	1.1	3.8	1.7
6	0.0	-15.00	105.00	120.00	6.4	2.5	-5.1	1.1	-2.8	1.6
7	0.0	-15.00	120.00	135.00	5.6	2.1	-5.1	1.1	-9.5	1.5
8	0.0	-15.00	135.00	150.00	18.2	2.0	7.3	1.1	2.3	1.5
9	0.0	-15.00	150.00	165.00	16.3	2.5	-11.4	1.3	-2.2	1.8
0	0.0	-15.00	165.00	180.00	-4.5	3.6	-8.2	1.4	-14.4	2.2
1	0.0	-15.00	180.00	195.00	4.2	3.8	7.7	1.4	5.6	2.2
2	0.0	-15.00	195.00	210.00	5.3	5.0	-3.2	1.4	-0.3	2.5
3	0.0	-15.00	210.00	225.00	-0.4	5.7	2.7	1.1	0.4	2.4
4	0.0	-15.00	225.00	240.00	-0.9	5.9	8.1	1.3	6.7	2.5
5	0.0	-15.00	240.00	255.00	0.1	5.9	-9.6	1.3	-5.1	2.4
6	0.0	-15.00	255.00	270.00	-0.8	5.9	-1.5	1.3	3.2	2.5
7	0.0	-15.00	270.00	285.00	-2.4	4.3	-10.8	1.3	-8.7	2.2
8	0.0	-15.00	285.00	300.00	7.5	4.2	2.1	1.3	-0.1	2.2
9	0.0	-15.00	300.00	315.00	-9.2	4.1	1.9	1.4	6.6	2.3
0	0.0	-15.00	315.00	330.00	-12.1	2.4	4.9	1.4	1.6	1.9
1	0.0	-15.00	330.00	345.00	-6.1	3.8	-0.6	1.5	4.6	2.4
2	0.0	-15.00	345.00	360.00	-1.2	3.9	5.1	1.4	7.6	2.2
3	15.00	0.0	0.0	15.00	8.9	3.0	-11.9	1.7	-6.6	2.1
4	15.00	0.0	15.00	30.00	-5.0	3.8	-0.3	1.7	-3.7	2.3
5	15.00	0.0	30.00	45.00	5.1	3.4	22.0	1.5	9.4	2.1
6	15.00	0.0	45.00	60.00	-16.2	2.6	-10.8	1.4	-3.9	1.8
7	15.00	0.0	60.00	75.00	-29.1	2.6	-12.9	1.5	-5.8	1.8
8	15.00	0.0	75.00	90.00	-26.0	2.8	-7.3	1.3	2.4	1.8
9	15.00	0.0	90.00	105.00	-5.7	2.4	6.0	1.2	5.6	1.6
0	15.00	0.0	105.00	120.00	7.2	3.0	7.0	1.1	2.9	1.8
1	15.00	0.0	120.00	135.00	24.9	3.2	0.5	1.2	6.1	1.9
2	15.00	0.0	135.00	150.00	4.7	3.0	-4.5	1.2	1.1	1.9
3	15.00	0.0	150.00	165.00	-5.1	3.8	11.5	1.3	1.5	2.2
4	15.00	0.0	165.00	180.00	8.0	5.0	2.5	1.5	6.5	2.6
5	15.00	0.0	180.00	195.00	-1.0	3.5	-4.1	1.4	4.9	2.2
6	15.00	0.0	195.00	210.00	6.0	4.7	-7.1	1.5	-8.9	2.5
7	15.00	0.0	210.00	225.00	-0.1	5.4	13.8	1.3	10.1	2.5
8	15.00	0.0	225.00	240.00	-0.8	5.9	-13.0	1.5	-6.0	2.6
9	15.00	0.0	240.00	255.00	-3.8	4.6	1.5	1.5	-3.8	2.4
0	15.00	0.0	255.00	270.00	1.3	4.2	7.3	1.4	-2.2	2.4
1	15.00	0.0	270.00	285.00	13.9	2.6	9.0	1.5	0.5	1.9
2	15.00	0.0	285.00	300.00	-3.6	3.0	-7.8	1.4	-2.0	2.0
3	15.00	0.0	300.00	315.00	-20.5	3.3	-3.3	1.5	-2.0	2.1
4	15.00	0.0	315.00	330.00	-6.7	3.1	2.8	1.5	-0.8	2.0
5	15.00	0.0	330.00	345.00	0.9	3.4	-9.5	1.6	-9.2	2.2
6	15.00	0.0	345.00	360.00	10.3	2.7	7.7	1.7	2.8	2.0
7	30.00	15.00	0.0	16.00	6.1	1.9	0.4	1.5	6.5	1.5
8	30.00	15.00	16.00	33.00	-0.3	3.1	-6.9	1.5	-3.9	2.0
9	30.00	15.00	33.00	49.00	3.4	2.9	-5.9	1.5	4.9	1.8
0	30.00	15.00	49.00	65.00	-10.1	3.3	0.8	1.6	-2.2	1.9
1	30.00	15.00	65.00	82.00	-13.5	2.1	22.6	1.4	3.9	1.5
2	30.00	15.00	82.00	98.00	-17.9	2.2	-1.3	1.2	-8.3	1.5

123	30.00	15.00	98.00	115.00	-14.5	2.5	-5.2	1.2	1.7	1.6
124	30.00	15.00	115.00	131.00	7.6	3.2	-11.5	1.3	-6.9	1.9
125	30.00	15.00	131.00	147.00	3.1	3.4	15.4	1.3	7.0	1.9
126	30.00	15.00	147.00	164.00	1.1	4.0	-6.7	1.4	-4.3	2.1
127	30.00	15.00	164.00	180.00	-6.1	3.4	-2.9	1.4	-0.1	2.1
128	30.00	15.00	180.00	196.00	-1.9	3.2	1.9	1.4	-5.5	2.0
129	30.00	15.00	196.00	213.00	6.7	3.0	6.9	1.3	6.3	1.9
130	30.00	15.00	213.00	229.00	-6.4	3.2	-11.6	1.4	-3.5	1.9
131	30.00	15.00	229.00	245.00	-13.6	3.4	24.0	1.4	9.7	2.0
132	30.00	15.00	245.00	262.00	-1.8	2.1	-11.2	1.4	-9.0	1.6
133	30.00	15.00	262.00	278.00	7.3	1.9	-11.0	1.4	9.4	1.5
134	30.00	15.00	278.00	295.00	-17.4	2.1	-7.1	1.4	-10.5	1.5
135	30.00	15.00	295.00	311.00	-24.6	2.3	12.2	1.3	8.5	1.6
136	30.00	15.00	311.00	327.00	-3.6	2.7	-4.2	1.4	0.2	1.8
137	30.00	15.00	327.00	344.00	3.0	2.6	5.1	1.4	3.4	1.7
138	30.00	15.00	344.00	360.00	0.8	2.3	-3.2	1.5	-5.4	1.7
139	45.00	30.00	0.0	19.00	9.4	1.9	-4.7	2.2	1.0	1.6
140	45.00	30.00	19.00	38.00	-1.7	2.5	13.8	2.2	-4.1	1.9
141	45.00	30.00	38.00	57.00	8.7	2.7	-4.7	2.2	5.5	2.0
142	45.00	30.00	57.00	76.00	-10.2	2.0	-8.5	2.0	-15.7	1.6
143	45.00	30.00	76.00	95.00	0.7	2.0	-14.2	1.8	2.7	1.6
144	45.00	30.00	95.00	114.00	-3.6	2.7	9.7	1.7	4.6	1.8
145	45.00	30.00	114.00	133.00	5.3	2.7	-0.4	1.8	3.9	1.9
146	45.00	30.00	133.00	152.00	1.6	2.5	4.5	2.1	-4.6	1.9
147	45.00	30.00	152.00	171.00	-5.9	3.5	-10.0	1.9	-3.2	2.3
148	45.00	30.00	171.00	189.00	-5.8	4.3	12.3	2.0	7.9	2.5
149	45.00	30.00	189.00	208.00	-5.6	3.5	-21.4	1.8	-10.7	2.2
150	45.00	30.00	208.00	227.00	-9.2	3.0	17.0	1.9	6.4	2.0
151	45.00	30.00	227.00	246.00	-12.3	1.7	-6.6	2.0	1.8	1.5
152	45.00	30.00	246.00	265.00	2.8	1.6	2.1	1.9	-1.3	1.3
153	45.00	30.00	265.00	284.00	-6.9	1.6	5.1	2.0	-0.3	1.3
154	45.00	30.00	284.00	303.00	-17.6	1.9	-2.2	2.1	-2.8	1.5
155	45.00	30.00	303.00	322.00	2.8	2.1	-6.1	2.0	-4.6	1.6
156	45.00	30.00	322.00	341.00	19.9	2.2	6.7	2.0	9.0	1.6
157	45.00	30.00	341.00	360.00	12.6	1.9	8.2	2.1	5.0	1.5
158	60.00	45.00	0.0	24.00	3.6	1.6	5.1	3.6	-3.7	1.4
159	60.00	45.00	24.00	48.00	0.9	2.0	-18.1	3.6	-2.3	1.7
160	60.00	45.00	48.00	72.00	-7.0	2.0	9.1	3.5	1.1	1.7
161	60.00	45.00	72.00	96.00	-21.4	1.8	9.9	2.9	1.4	1.5
162	60.00	45.00	96.00	120.00	-16.0	2.6	-12.6	2.2	3.1	1.9
163	60.00	45.00	120.00	144.00	-1.5	3.0	14.8	2.4	0.4	2.2
164	60.00	45.00	144.00	168.00	7.5	4.4	-4.3	3.2	-0.7	2.8
165	60.00	45.00	168.00	192.00	0.1	2.7	-14.8	3.2	-5.3	2.2
166	60.00	45.00	192.00	216.00	7.2	3.0	28.8	2.7	8.5	2.1
167	60.00	45.00	216.00	240.00	1.9	2.3	-22.2	2.5	-1.5	1.8
168	60.00	45.00	240.00	264.00	-2.4	1.6	14.5	2.6	-0.4	1.4
169	60.00	45.00	264.00	288.00	-23.8	1.7	-11.2	3.0	-5.5	1.4
170	60.00	45.00	288.00	312.00	-6.6	2.1	-5.1	3.1	2.0	1.7
171	60.00	45.00	312.00	336.00	12.6	3.4	14.4	2.8	6.1	2.2
172	60.00	45.00	336.00	360.00	8.2	2.5	-8.7	3.2	-3.5	2.0
173	75.00	60.00	0.0	40.00	1.6	2.3	-7.8	2.4	-7.0	1.9
174	75.00	60.00	40.00	80.00	-6.2	3.5	-9.8	2.5	0.5	2.2
175	75.00	60.00	80.00	120.00	-20.0	2.1	10.0	2.3	-2.7	1.7
176	75.00	60.00	120.00	160.00	1.1	2.5	-5.8	2.3	-1.7	1.7
177	75.00	60.00	160.00	200.00	7.5	2.5	2.7	2.3	2.8	1.8
178	75.00	60.00	200.00	240.00	5.3	1.8	2.1	2.3	6.8	1.5
179	75.00	60.00	240.00	280.00	-26.8	2.0	-11.1	2.3	-14.0	1.5
180	75.00	60.00	280.00	320.00	-7.5	2.4	8.1	2.2	3.1	1.8
181	75.00	60.00	320.00	360.00	14.4	3.5	11.4	2.1	12.0	2.0
182	90.00	75.00	0.0	120.00	-1.7	4.5	8.6	1.1	7.1	1.6
183	90.00	75.00	120.00	240.00	-8.1	3.2	-8.5	1.1	-6.6	1.5
184	90.00	75.00	240.00	360.00	2.0	3.1	0.1	0.9	-0.1	1.3

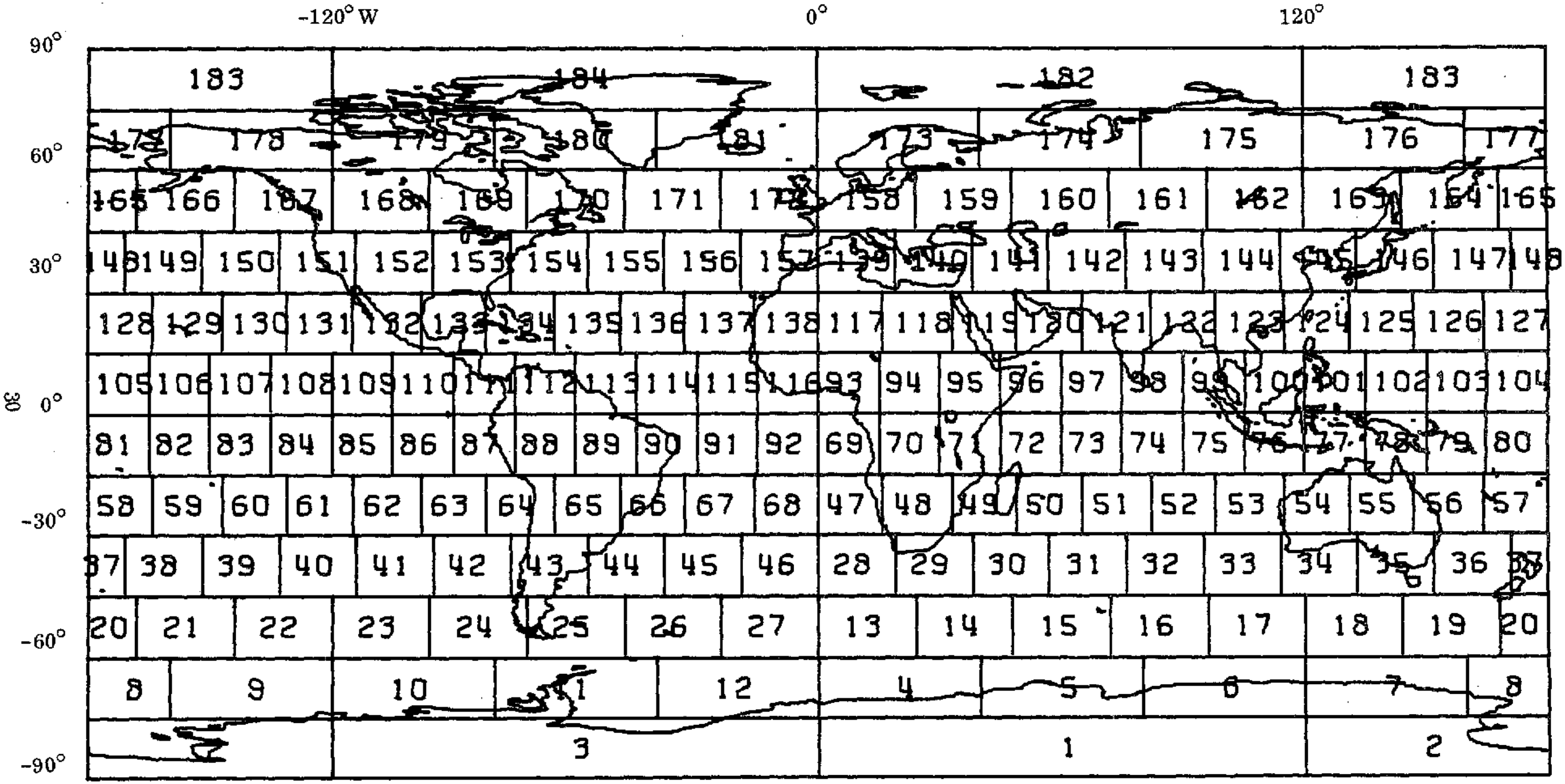

Figure One
Block Square Numbers for 15° Equal Area Blocks

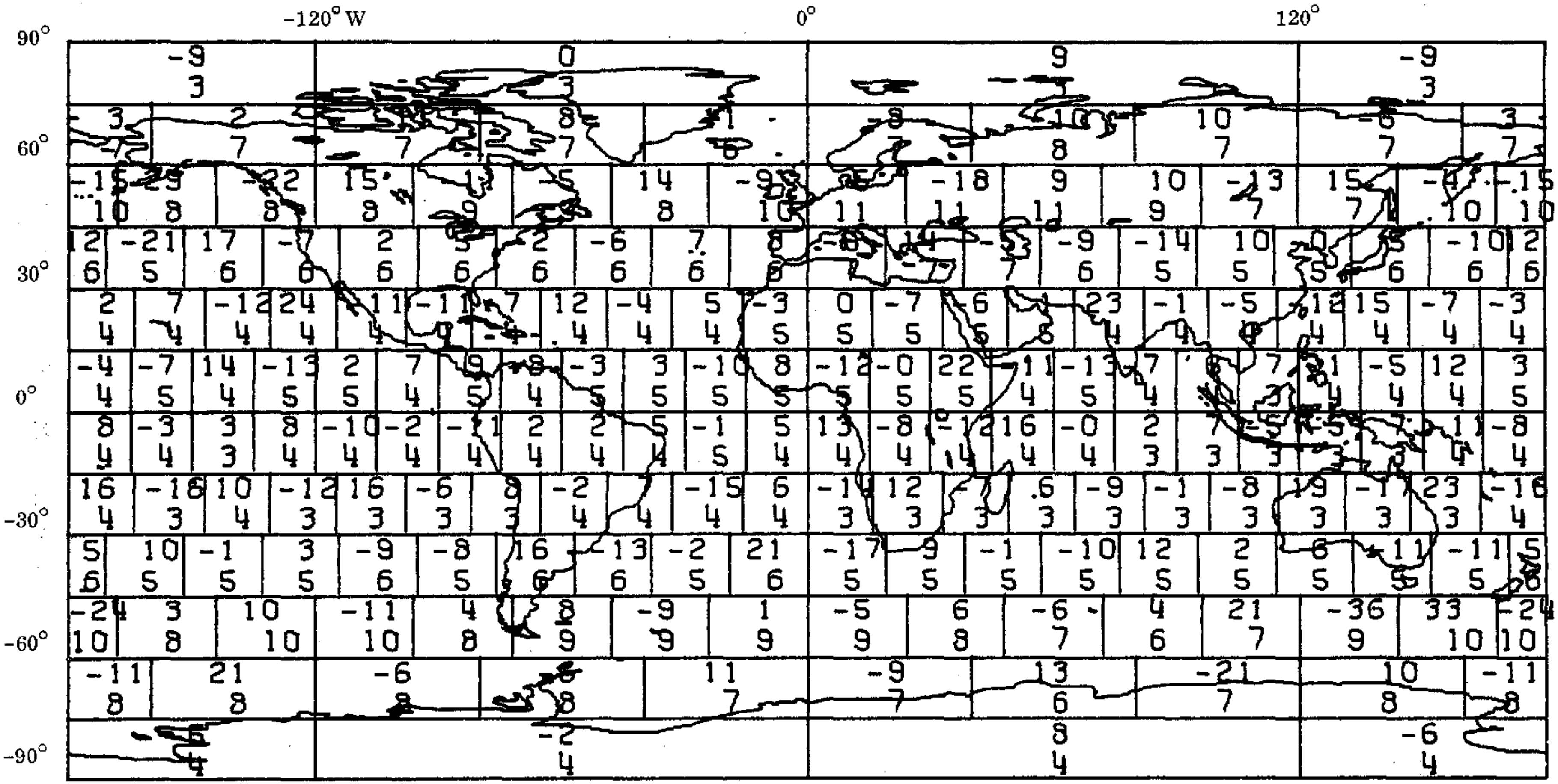

Figure Two
Anomalies (upper figure) and Standard Deviations
From 29 Arc Satellite Solution
(mgals)

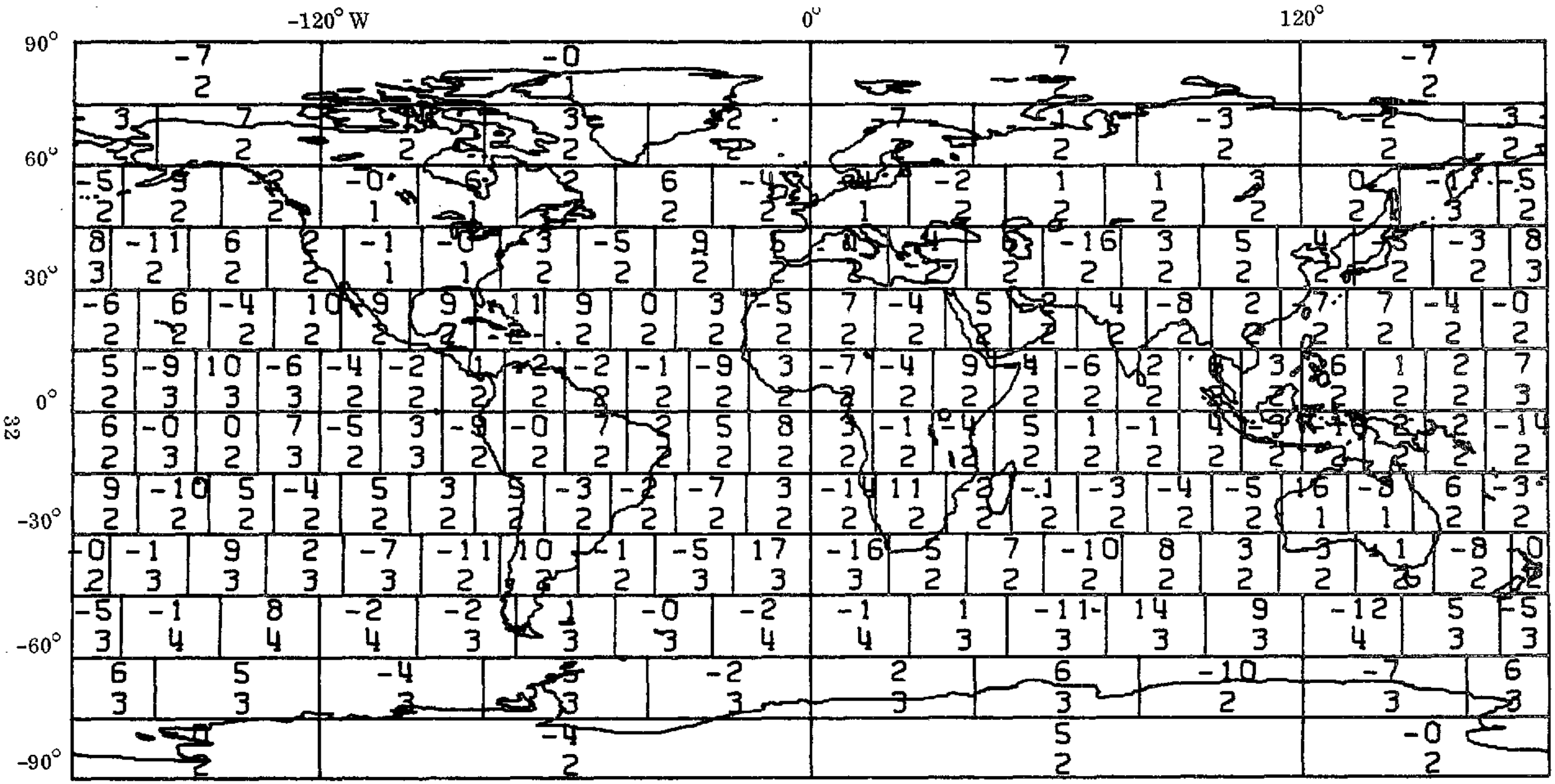

Figure Three

Anomalies (upper figure) and their Standard Deviations
From 29 Arc Combination Solution
(mgals)

as obtained from the 29 arc satellite and the 29 arc combination solution. (In the 29 arc satellite solution the standard deviations given in the figure have been obtained by multiplying the solution standard deviations by three.)

In Table 8 we give potential coefficient solutions of interest. The first set of coefficients is the input or reference set of coefficients. These values are repeated from Table 4. The second set are those coefficients implied by the 15° terrestrial anomaly field. The third and fourth sets are those coefficients implied by the 29 arc satellite and 29 arc combination solutions computed using (63). Finally the GEM 3 coefficients are given for comparison purposes.

The geoid undulations implied by the 29 arc combination solution are shown in Figure 4. These undulations have been computed from the following equation:

$$N = R \sum_{\ell=2}^{12} \sum_{m=0}^{\ell} (\overline{C}^{*}_{\ell m} \cos m\lambda + \overline{S}_{\ell m} \sin m\lambda)\, \overline{P}_{\ell m}(\sin \varphi') \tag{67}$$

with a reference flattening of 1/298.256.

It is also of interest to consider the anomaly degree variance implied by the several solutions. These values may be computed from:

$$\sigma_{\ell}^{2}(\Delta g) = \gamma^{2} \sum_{m=0}^{\ell} (\overline{C}^{*2}_{\ell m} + \overline{S}^{2}_{\ell m}) \tag{68}$$

Such values are shown in Table 9 as computed from:

1. a combination solution of gravimetric data using potential coefficients as described by Rapp (1973);

2. the coefficients of the SAO Standard Earth II;

3. the coefficients of the 29 arc satellite solution, and

4. the coefficients of the 29 arc combination solution.

No distortion of the anomaly degree variances as found from the direct combination solution appears evident.

Table 8

Potential Coefficient Information

Input		Gravity Only		29 Arc Sat Only		29 Arc Comb		GEM 3	
C(I)	S(I)	C(T)	S(T)	C(S)	S(S)	C(C)	S(C)	C(G)	S(G)
-484.167		-484.467		-484.160		-484.163		-484.172	
2.380	-1.351	2.774	-0.832	2.450	-1.360	2.444	-1.361	2.425	-1.386
0.959		0.373		0.956		0.955		0.958	
1.936	0.266	1.340	0.170	2.004	0.194	1.995	0.208	2.017	0.251
0.735	-0.539	0.974	-0.442	0.872	-0.698	0.908	-0.695	0.914	-0.624
0.561	1.621	0.828	1.222	0.668	1.376	0.685	1.388	0.720	1.420
0.531		0.543		0.535		0.536		0.547	
-0.572	-0.469	-0.366	-0.228	-0.542	-0.475	-0.542	-0.473	-0.532	-0.444
0.330	0.662	0.398	0.304	0.338	0.662	0.336	0.668	0.354	0.664
0.852	-0.191	0.796	-0.260	0.936	-0.205	0.905	-0.205	0.976	-0.220
-0.053	0.230	-0.055	0.254	-0.180	0.298	-0.183	0.338	-0.181	0.312
0.069		-0.018		0.067		0.069		0.068	
-0.079	-0.103	-0.218	-0.053	-0.032	-0.090	-0.043	-0.074	-0.069	-0.082
0.630	-0.232	0.457	-0.062	0.663	-0.375	0.663	-0.317	0.657	-0.319
-0.521	0.007	-0.310	-0.138	-0.474	-0.194	-0.456	-0.195	-0.467	-0.278
-0.265	0.064	-0.027	-0.025	-0.352	-0.058	-0.284	0.010	-0.321	0.025
0.156	-0.593	0.137	-0.518	0.131	-0.587	0.116	-0.584	0.148	-0.678
-0.139		-0.009		-0.136		-0.136		-0.162	
-0.047	-0.027	-0.071	-0.113	-0.066	0.038	-0.070	0.041	-0.089	-0.021
0.069	-0.366	0.166	-0.209	0.100	-0.370	0.114	-0.348	0.068	-0.370
-0.054	0.031	-0.095	-0.065	-0.020	0.018	-0.029	0.019	0.023	-0.026
-0.044	-0.518	-0.146	-0.340	-0.069	-0.462	-0.096	-0.466	-0.109	-0.458
-0.313	-0.458	-0.321	-0.417	-0.325	-0.469	-0.341	-0.459	-0.303	-0.505
-0.040	-0.155	-0.024	-0.152	-0.054	-0.248	-0.052	-0.242	0.041	-0.221
0.093		0.077		0.097		0.095		0.092	
0.197	0.156	0.190	0.137	0.225	0.143	0.226	0.128	0.252	0.131
0.364	0.163	0.354	0.079	0.341	0.083	0.367	0.108	0.336	0.080
0.250	0.018	0.161	-0.084	0.243	-0.131	0.202	-0.157	0.265	-0.222
-0.152	-0.102	-0.115	-0.144	-0.201	-0.137	-0.165	-0.142	-0.313	-0.087
0.076	0.054	-0.008	0.034	0.077	0.071	0.030	0.074	-0.010	0.056
-0.209	0.063	-0.203	0.105	-0.267	0.105	-0.232	0.086	-0.332	0.156
0.055	0.097	-0.021	0.025	0.115	0.038	0.066	0.038	0.065	0.038
0.029		-0.019		0.031		0.031		0.062	
-0.076	0.065	-0.081	0.054	0.002	0.043	-0.001	0.028	0.028	0.094
0.026	0.039	0.075	0.122	0.059	0.045	0.079	0.066	0.048	0.065
-0.037	0.004	0.006	0.017	-0.085	-0.036	-0.072	-0.021	-0.024	-0.083
-0.212	-0.012	-0.177	-0.003	-0.220	0.020	-0.209	-0.003	-0.253	0.069
-0.053	0.118	-0.022	0.058	-0.076	0.015	-0.085	0.041	-0.096	0.086
-0.017	0.318	-0.063	0.157	-0.009	0.247	-0.029	0.208	-0.035	0.307
-0.009	0.031	0.033	0.082	-0.028	0.067	0.013	0.108	0.052	0.071
-0.248	0.102	-0.119	0.039	-0.159	0.065	-0.152	0.061	-0.093	0.097
0.023		0.096		0.018		0.021		0.030	
0.117	0.012	0.157	0.029	0.178	0.043	0.162	0.046	0.161	0.002
-0.004	0.035	0.034	-0.034	-0.007	0.011	0.007	0.018	0.024	-0.018
0.0	0.0	-0.093	0.011	-0.073	-0.027	-0.076	0.019	-0.149	-0.152
0.0	0.0	-0.041	0.035	-0.003	0.056	-0.051	0.033	0.003	0.034
0.0	0.0	-0.040	0.026	0.020	0.040	0.005	0.069	-0.020	-0.068
0.0	0.0	0.012	0.046	0.044	0.106	0.024	0.053	0.090	0.229
0.0	0.0	-0.043	0.013	-0.076	-0.091	-0.049	-0.047	-0.057	-0.028
0.0	0.0	0.124	0.021	0.034	0.021	0.065	0.029	0.181	-0.030
0.185	0.210	0.113	0.116	0.163	0.142	0.152	0.127	-0.035	0.076
0.077		0.011		0.078		0.078		0.040	
0.105	-0.126	0.063	-0.101	0.072	-0.129	0.066	-0.105	0.076	-0.180

L	M	C(I)	S(I)	C(T)	S(T)	C(S)	S(S)	C(C)	S(C)	C(G)	S(G)
10	2	-0.105	-0.042	-0.060	-0.052	-0.045	-0.037	-0.047	-0.031	-0.047	-0.041
10	3	-0.065	0.030	-0.021	-0.035	-0.076	-0.082	-0.039	-0.064	-0.041	-0.121
10	4	-0.074	-0.111	-0.074	-0.068	-0.140	-0.158	-0.096	-0.116	-0.098	-0.110
10	5	0.0	0.0	-0.004	0.002	-0.054	-0.093	-0.012	-0.058	-0.110	-0.013
10	6	0.0	0.0	-0.001	-0.031	-0.093	-0.127	-0.037	-0.080	0.004	-0.123
10	7	0.0	0.0	0.066	0.019	0.0	0.076	0.014	0.064	-0.019	-0.037
10	8	0.0	0.0	-0.009	-0.045	-0.039	-0.057	0.015	-0.050	0.048	-0.136
10	9	0.104	-0.064	0.132	-0.025	0.127	-0.039	0.124	-0.012	0.116	-0.066
10	10	0.0	0.0	0.010	0.005	0.031	0.027	0.011	0.033	0.064	-0.009
11	0	-0.042		-0.053		-0.038		-0.040		-0.056	
11	1	-0.053	0.015	-0.043	0.010	-0.053	0.065	-0.048	0.046	-0.016	0.033
11	2	0.0	0.0	-0.005	-0.016	-0.018	-0.041	-0.015	-0.036	0.036	-0.113
11	3	0.0	0.0	-0.031	-0.009	0.0	-0.058	-0.029	-0.036	-0.011	-0.119
11	4	0.0	0.0	-0.050	-0.038	0.004	-0.004	-0.039	-0.029	0.019	-0.077
11	5	0.0	0.0	0.020	0.013	0.055	0.046	0.013	0.027	0.027	0.025
11	6	0.0	0.0	0.026	-0.028	-0.013	0.039	0.011	-0.003	-0.034	0.060
11	7	0.0	0.0	0.029	-0.057	0.021	-0.047	0.022	-0.069	0.011	-0.116
11	8	0.0	0.0	-0.031	0.045	0.043	0.096	-0.006	0.071	-0.027	0.034
11	9	0.0	0.0	-0.015	0.013	-0.001	-0.027	0.004	-0.009	-0.014	0.047
11	10	0.0	0.0	-0.052	0.003	0.026	-0.071	0.017	-0.021	-0.109	0.005
11	11	0.027	0.056	0.031	0.002	0.032	-0.005	0.030	0.022	0.085	-0.022
12	0	0.008		-0.012		0.009		0.009		0.046	
12	1	-0.163	-0.071	-0.046	-0.070	-0.127	-0.076	-0.107	-0.096	-0.066	-0.015
12	2	-0.103	-0.005	-0.097	-0.007	-0.100	0.005	-0.101	0.003	-0.041	0.037
12	3	0.0	0.0	0.034	-0.030	0.026	0.088	0.035	0.023	0.112	0.086
12	4	0.0	0.0	-0.013	0.011	-0.031	-0.011	-0.019	-0.003	-0.019	-0.013
12	5	0.0	0.0	0.022	0.002	0.061	-0.019	0.052	-0.018	0.030	-0.009
12	6	0.0	0.0	0.009	0.020	-0.044	0.031	-0.021	0.032	0.061	-0.010
12	7	0.0	0.0	-0.038	0.009	-0.022	0.033	-0.031	0.025	-0.022	0.011
12	8	0.0	0.0	0.036	0.011	0.021	-0.044	0.036	-0.023	-0.034	-0.027
12	9	0.0	0.0	0.006	-0.004	-0.046	0.050	-0.015	0.023	0.034	0.033
12	10	0.0	0.0	0.011	0.010	-0.011	0.012	-0.014	0.011	-0.022	0.057
12	11	-0.054	-0.311	-0.043	-0.121	-0.066	-0.090	-0.065	-0.154	0.009	0.034
12	12	-0.033	-0.005	-0.033	-0.004	-0.023	0.001	-0.023	0.004	-0.012	0.005

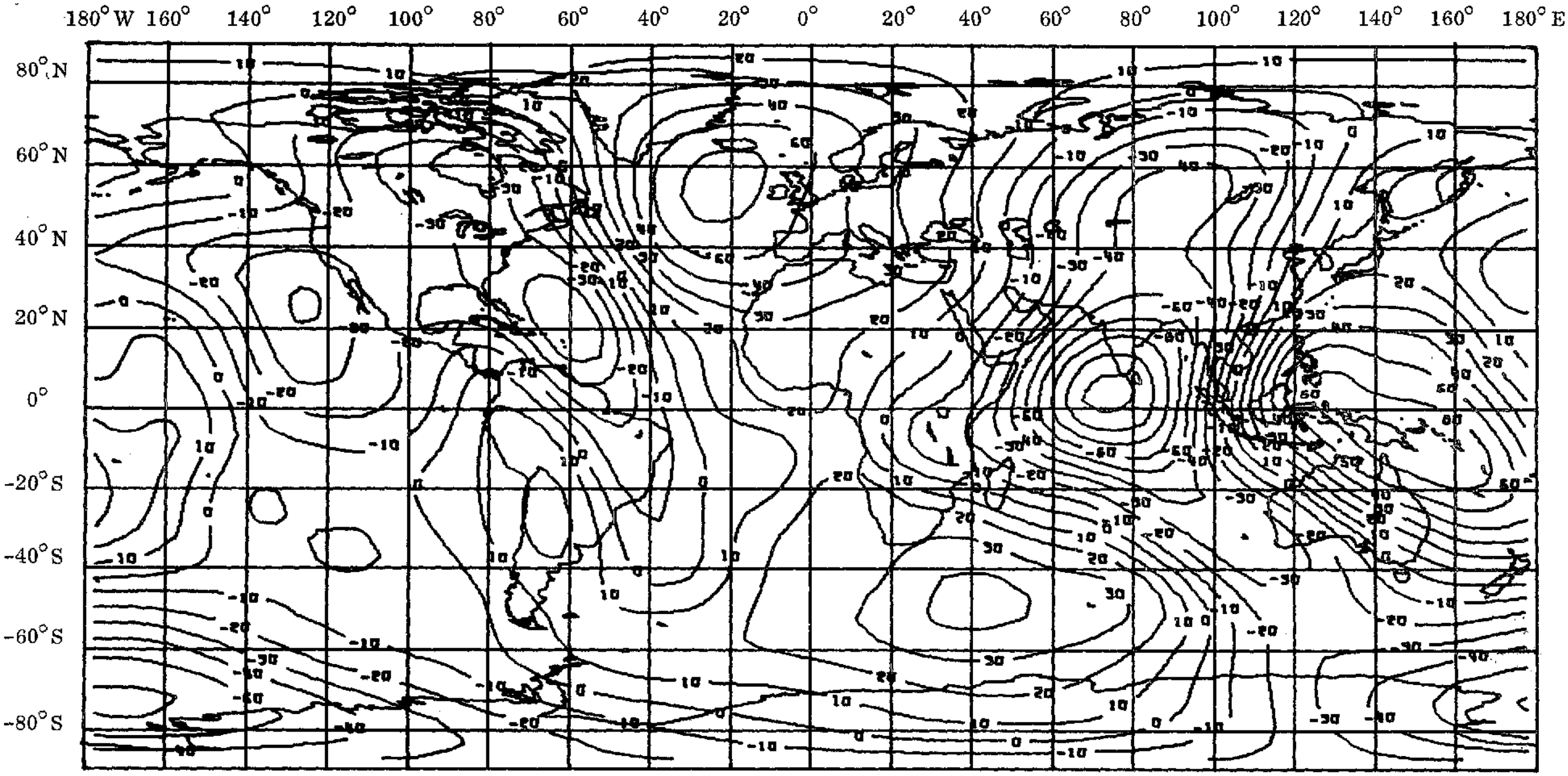

Figure Four
Geoid Undulations from 29 Arc Combination Solutions
Reference Flattening =1/298.256

Table 9

Anomaly Degree Variances
(mgal^2)

ℓ	Rapp (1973)	SE II	29 arc sat	29 arc comb
2	7.5	7.4	7.5	7.5
3	33.9	33.0	32.9	33.2
4	19.2	20.0	18.8	18.6
5	21.6	17.8	20.6	18.9
6	18.9	15.7	18.7	18.6
7	18.8	15.5	15.4	14.1
8	10.4	6.7	7.9	7.2
9	11.1	12.7	7.4	5.9
10	11.4	12.9	12.0	6.8
11	8.4	12.2	4.0	2.5
12	4.8	5.1	8.2	8.2

We next give in Table 10 the X, Y, Z station coordinates found from the 29 arc satellite and the 29 arc combination solution. In addition we give the difference between the specific solution and the coordinates of the GEM 4 solution. The last line for each station gives the root mean square coordinate difference between the solutions. We summarize these differences in Table 11 where we also show the shift between the initial coordinate values and the final adjusted value.

Table 11

RMS Coordinate Shifts (Adjusted vs Initial) (Δ_1)
and RMS Coordinate Differences (Adjusted vs GEM 4) (Δ_2)

	Δ_1		Δ_2	
Station	29 arc sat	29 arc comb	29 arc sat	29 arc comb
9001	16.5m	15.8m	2.2 m	1.4 m
9002	10.0	12.2	8.5	4.4
9004	18.1	17.7	4.0	2.6
9006	18.6	16.5	4.3	2.3
9007	10.8	10.2	2.7	2.9
9012	21.7	24.8	6.9	9.0
9023	16.8	16.0	7.0	5.8

Table 10

Rectangular Coordinates for 7 Stations with Differences from GEM 4 Coordinates (meters)

STATION	SAT29	COM29	DIF1	DIF2
9001	-1535740.88	-1535740.75	0.36	0.49
9001	-5166999.94	-5167000.17	1.31	1.08
9001	3401050.12	3401051.06	-1.75	-0.81
9001			2.22	1.44
9002	5056127.81	5056128.13	-4.16	-3.84
9002	2716529.02	2716522.93	7.37	1.28
9002	-2775770.75	-2775772.18	-0.42	-1.85
9002			8.47	4.45
9004	5105590.36	5105591.75	-3.74	-2.35
9004	-555223.21	-555223.08	-1.07	-0.94
9004	3769677.27	3769675.67	1.08	-0.52
9004			4.04	2.58
9006	1018195.83	1018195.71	1.51	1.39
9006	5471106.27	5471108.58	-3.96	-1.65
9006	3109630.14	3109630.06	0.96	0.88
9006			4.35	2.33
9007	1942788.83	1942788.97	-1.46	-1.32
9007	-5804088.47	-5804090.02	-0.99	-2.54
9007	-1796924.47	-1796926.66	1.98	-0.21
9007			2.65	2.87
9012	-5466048.24	-5466045.93	5.12	7.43
9012	-2404293.67	-2404293.75	4.52	4.44
9012	2242187.07	2242184.00	0.79	-2.28
9012			6.88	8.95
9023	-3977779.01	-3977779.49	5.80	5.32
9023	3725104.47	3725105.83	-2.52	-1.16
9023	-3303008.27	-3303009.16	3.01	2.12
9023			7.01	5.84

We conclude from examination of Table 11 that the station coordinates found from the two specific solutions of this paper are in reasonably good agreement with those found from the GEM 4 solutions. In fact the 29 arc combination solution shows better agreement then the 29 arc satellite solution. This would indicate that the addition of the terrestrial gravity material was helpful in station coordinate determinations.

10. Conclusions

The purpose of this report has been to detail a method for solving directly for gravity anomalies using satellite observations, and in combination with observed terrestrial anomalies. The method was tested using approximately 20,000 optical satellite observations. The results (both for anomalies) and station coordinates indicate that the proposed method works and may be used to refine our knowledge of the earth's gravitational field.

Since this test was made with 184 15° blocks and a limited sample of satellite data, we might continue the study adding more anomaly blocks and satellite data. A 15° discrete anomaly block field is roughly equivalent to a spherical harmonic expansion to degree 12, which is about the degree of potential coefficients that can be determined from current satellite data using the more conventional techniques. Thus, at this time, I would not suggest taking smaller anomaly blocks to solve for using satellite data currently available. We could, however, process more data. However, this would be expensive and probably not worth the effort since conventional analysis has already been carried out. (For a 7 day arc, the computer time necessary for the orbit integration, formation of the complete normal equations, etc., is approximately 35 minutes (on the average) when our IBM 370/165 is used with 184 15° blocks and station coordinates. Increasing the number of unknown stations would somewhat increase this running time but not as much as would result if the number of anomaly blocks were increased. Suppose for argument, then, that each 7 day arc being processed takes 40 minutes. In the GEM 5 solution (Richardson and Lerch, 1974) 350 7 day arcs were processed. This number of arcs would then take our method 233 hours plus any additional time needed for orbit convergence, etc. Thus, it would not be unreasonable to estimate 300 hours as the computational time on our IBM 370/165 to repeat the GEM 5 solution. The cost would be approximately $150,000.)

The beauty of the proposed method lies in several areas:

1. The gravitational field parameters (i.e. the gravity anomalies) are directly related to an averaging of terrestrial gravity measurements. This contrast with potential coefficients or surface density values which are integrals of the gravity measurements.

2. We can use the method to solve for gravity anomalies in regional or local areas assuming the sufficiently precise satellite data is available. And such data is expected from satellite to satellite tracking, altimeter data and possibly gravity gradient devices. A study (being carried out by D. P. Hajela) is nearing completion bearing on the recovery of gravity anomalies in local areas from satellite to satellite tracking data.

Finally we should mention that in the implementation of this method, the anomalies derived from the satellite data alone, will refer to the Bjerhammar sphere. Consequently, when a combination solution is carried out, the terrestrial anomalies, should be reduced from being surface free-air anomalies to free-air anomalies referring to the Bjerhammar sphere (located in the interior of the earth). Such reductions are negligible within the current accuracy of our knowledge of the terrestrial gravity field in 15° blocks.

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Appendix

Table A contains the specific orbital information, after several inner iterations, for the 39 arcs used in this study.

Table A

ARC NUMBER 1
ANNA 620601

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
660102 0	4.264	1.100
X(METERS)	Y(METERS)	Z(METERS)
-5764417.28	2301944.28	-4227857.28
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-4458.07	-4302.23	3831.24

ARC NUMBER 2
BEB 640641

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
670226 0	1.005	1.100
X(METERS)	Y(METERS)	Z(METERS)
540032.18	-7363601.41	-221737.99
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
1317.38	-20.63	7216.70

ARC NUMBER 3
BEC 650321

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
670404 0	3.010	1.100
X(METERS)	Y(METERS)	Z(METERS)
15722.68	-6604800.11	3215968.83
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
6235.39	-1862.74	-3616.00

ARC NUMBER 4
COURIER 600131

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
661231 0	3.036	1.100
X(METERS)	Y(METERS)	Z(METERS)
-6517757.95	632764.28	3493936.89
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-378.79	-7332.07	374.19

ARC NUMBER 5
DIC 670111

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
670317 0	1.426	1.100
X(METERS)	Y(METERS)	Z(METERS)
-519218.99	6013084.54	4341945.80
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-6978.72	1106.94	-1725.08

ARC NUMBER 6
GEOS A 650891

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
660216 70000	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
5810635.12	3122538.02	4338905.91
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
327.24	5135.58	-5011.92

ARC NUMBER 7
GEOS B 680021

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
680414 0	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
2127520.18	1817051.79	7153944.54
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
6954.35	-1141.43	-1528.49

ARC NUMBER 8
OSCAR 660051

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
660408 0	1.541	1.100
X(METERS)	Y(METERS)	Z(METERS)
-21353.28	-468455.06	-7432004.60
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
932.79	7229.49	-286.42

ARC NUMBER 9
OVI-2 650781

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
661111 0	0.459	1.100
X(METERS)	Y(METERS)	Z(METERS)
5873849.13	7530018.31	-914842.62
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
4344.91	-2251.13	3344.97

ARC NUMBER 10
ANNA 620601

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
651222 0	2.383	1.100
X(METERS)	Y(METERS)	Z(METERS)
-4868359.82	-5623193.85	-543352.19
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
3307.32	-3381.73	5610.25

ARC NUMBER 11
BEB 640641

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
670316 0	4.389	1.100
X(METERS)	Y(METERS)	Z(METERS)
1961932.87	2803105.49	6453186.70
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
1172.73	6614.69	-3130.99

ARC NUMBER 12
BEC 650321

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
660325 180000	7.509	1.100
X(METERS)	Y(METERS)	Z(METERS)
-3226654.08	6497087.81	-2286315.44
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-4427.12	-3840.19	-4169.38

ARC NUMBER 13
COURIER 600131

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670707 0	3.622	1.100
X(METERS)	Y(METERS)	Z(METERS)
-7029599.25	-608597.07	-2196533.06
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-349.24	-6862.74	2703.93

ARC NUMBER 14
DIC 670111

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670224 0	1.144	1.100
X(METERS)	Y(METERS)	Z(METERS)
3342651.92	6541817.16	-1060857.04
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-4548.45	3481.22	4507.89

ARC NUMBER 15
DID-7 670141

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670528 40000	1.765	1.100
X(METERS)	Y(METERS)	Z(METERS)
-5345073.66	3388390.86	4236912.79
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-2763.23	-6179.52	2550.97

ARC NUMBER 16
GEOS A 650891

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
651231 210000	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
3037967.94	-5162149.54	-5795899.16
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
5993.70	-699.36	3138.22

ARC NUMBER 17
GEOS B 680021

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
681006 0	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
-230588.60	3531574.10	-7115431.04
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
3931.67	5171.05	2520.26

ARC NUMBER 18
OSCAR 660051

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
660415 0	2.664	1.100
X(METERS)	Y(METERS)	Z(METERS)
592974.23	4238891.00	-5894617.47
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
761.73	5921.84	4471.26

ARC NUMBER 19
OVI-2 650781

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
661104 0	0.513	1.100
X(METERS)	Y(METERS)	Z(METERS)
-6041999.88	2873001.51	-3446650.10
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
3256.60	6054.81	-3270.61

ARC NUMBER 20
ANNA 620601

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
651211 0	2.178	1.100
X(METERS)	Y(METERS)	Z(METERS)
3100140.47	-5880846.61	3424685.83
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
3598.16	4483.28	4525.64

ARC NUMBER 21
BEC 650321

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
660423 0	6.598	1.100
X(METERS)	Y(METERS)	Z(METERS)
-4004511.47	-4034145.83	4672105.24
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
5952.44	-4280.67	1212.35

ARC NUMBER 22
COURIER 600131

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670108 0	3.981	1.100
X(METERS)	Y(METERS)	Z(METERS)
2742353.36	6970624.82	1152032.26
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-6133.17	1923.81	3245.19

ARC NUMBER 23
DID-7 670141

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670514 0	1.788	1.100
X(METERS)	Y(METERS)	Z(METERS)
-1598253.22	5956535.93	-3749158.66
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-7190.10	270.63	2519.01

ARC NUMBER 24
GEOS A 650891

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
661115 0	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
-7308606.91	-358001.30	-4626778.84
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
3001.51	-3795.70	-4393.10

ARC NUMBER 25
GEOS B 680021

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
680915 0	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
-6311185.50	-3951842.35	1100320.95
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-2091.04	1079.15	-6978.12

ARC NUMBER 26
OSCAR 660051

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
660401 13000	0.327	1.100
X(METERS)	Y(METERS)	Z(METERS)
-940530.22	-7507320.70	318690.65
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-11.56	-393.69	-7169.84

ARC NUMBER 27
OVI-2 650781

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
661118 0	0.662	1.100
X(METERS)	Y(METERS)	Z(METERS)
6875416.28	-3136588.57	5428186.38
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-3050.33	-5289.82	-640.05

ARC NUMBER 28
BEC 650321

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
660314 20000	0.595	1.100
X(METERS)	Y(METERS)	Z(METERS)
-6550077.84	-128480.45	-3386375.53
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
2047.00	-6242.69	-3441.10

ARC NUMBER 29
COURIER 600131

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670127 0	0.817	1.100
X(METERS)	Y(METERS)	Z(METERS)
6932617.30	-941332.72	2245112.99
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
104.98	6913.64	2709.44

ARC NUMBER 30
DID-7 670141

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670507 0	1.580	1.100
X(METERS)	Y(METERS)	Z(METERS)
-456026.80	-6143323.90	5035605.69
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
6914.05	178.97	64.18

ARC NUMBER 31
GEOS A 650891

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
660709 0	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
-887580.99	8170773.40	-1348816.21
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-3337.67	-1740.50	-5674.87

ARC NUMBER 32
GEOS B 680021

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
680608 0	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
2617701.95	1677438.95	-6840379.69
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-1688.65	-6837.45	-2172.05

ARC NUMBER 33
OSCAR 660051

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
660422 0	3.265	1.100
X(METERS)	Y(METERS)	Z(METERS)
965119.55	7016440.65	-1591207.95
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
192.60	1691.68	7294.59

ARC NUMBER 34
BEC 650321

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670317 0	1.436	1.100
X(METERS)	Y(METERS)	Z(METERS)
-2178548.75	5238253.79	-4957781.88
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-6690.29	-2804.49	246.48

ARC NUMBER 35
COURIER 600131

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670714 0	2.604	1.100
X(METERS)	Y(METERS)	Z(METERS)
-321720.11	-6620933.59	3540068.42
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
7243.00	-233.67	451.41

ARC NUMBER 36
DID-7 670141

EPOCH OF ELEMENTS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
YRMODD HHMMSS		
670305 0	1.973	1.100
X(METERS)	Y(METERS)	Z(METERS)
-5913747.63	2753311.12	4981459.59
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-1717.67	-6369.21	1124.31

ARC NUMBER 37
GEOS A 650891

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
660925 0	0.0	1.100
X(METERS)	Y(METERS)	Z(METERS)
-3931190.10	2651463.70	6188672.69
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-1021.05	-6642.06	2772.85

ARC NUMBER 38
BEC 650321

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
670415 120000	3.382	1.100
X(METERS)	Y(METERS)	Z(METERS)
-1715286.66	6789909.08	2978466.17
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-5267.77	-3140.58	3748.24

ARC NUMBER 39
COURIER 600131

EPOCH OF ELEMENTS YRMODD HHMMSS	DRAG COEFFICIENT	SOLAR REFLECTIVITY
670623 0	1.780	1.100
X(METERS)	Y(METERS)	Z(METERS)
6606542.89	379803.27	3393486.13
XDOT(M/S)	YDOT(M/S)	ZDOT(M/S)
-1010.51	7211.75	895.84